

In **Pre-Kindergarten**, instructional time should focus on two critical areas: (1) developing an understanding of whole numbers using concrete materials, including concepts of correspondence, counting, cardinality, and comparison; (2) describing shapes in their environment. More learning time in Pre-Kindergarten should be devoted to developing the concept of number than to other topics.

(1) Students develop an understanding of the meanings of whole numbers and recognize the number of objects in small groups by counting – the first and most basic mathematical algorithm. They understand that number words refer to quantity. They use one-to-one correspondence to solve problems by matching sets and comparing number amounts and in counting objects to 10. They understand that the last word that they state in counting tells “how many” and they count to determine number amounts and compare quantities (using language such as “more than” and “less than”).

(2) Students describe their physical world using geometric ideas (e.g., shape and special relations) and vocabulary. They identify and name basic two-dimensional shapes, such as triangles, rectangles, squares, and circles. They use basic shapes and spatial reasoning to model objects in their environment.

<p><b>Grade PK Overview Counting and Cardinality</b></p> <ul style="list-style-type: none"> <li>• Know number names and the count sequence.</li> <li>• Count to tell the number of objects.</li> <li>• Compare numbers.</li> </ul> <p><b>Operations and Algebraic Thinking</b></p> <ul style="list-style-type: none"> <li>• Understand addition as adding to, and understand subtraction as taking from.</li> </ul> <p>Understand simple patterns.</p>	<p><b>Measurement and Data</b></p> <ul style="list-style-type: none"> <li>• Describe and compare measurable attributes.</li> <li>• Sort objects and count the number of objects in each categories.</li> </ul> <p><b>Geometry</b></p> <ul style="list-style-type: none"> <li>• Identify and describe shapes (squares, circles, triangles, rectangles).</li> <li>• Analyze, compare, and sort objects.</li> </ul>
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In **Kindergarten**, instructional time should focus on two critical areas: (1) representing and comparing whole numbers, initially with sets of objects; (2) describing shapes and space. More learning time in Kindergarten should be devoted to number than to other topics.

1. Students use numbers, including written numerals, to represent quantities and to solve quantitative problems, such as counting objects in a set; counting out a given number of objects; comparing sets or numerals; and modeling simple joining and separating situations with sets of objects, or eventually with equations such as  $5 + 2 = 7$  and  $7 - 2 = 5$ . (Kindergarten students should see addition and subtraction equations, and student writing of equations in kindergarten is encouraged, but it is not required.) Students choose, combine, and apply effective strategies for answering quantitative questions, including quickly recognizing the cardinalities of small sets of objects, counting and producing sets of given sizes, counting the number of objects in combined sets, or counting the number of objects that remain in a set after some are taken away.

2. Students describe their physical world using geometric ideas (e.g., shape, orientation, spatial relations) and vocabulary. They identify, name, and describe basic two-dimensional shapes, such as squares, triangles, circles, rectangles, and hexagons, presented in a variety of ways (e.g., with different sizes and orientations), as well as three-dimensional shapes such as cubes, cones, cylinders, and spheres. They use basic shapes and spatial reasoning to model objects in their environment and to construct more complex shapes.

<p><b>Grade K Overview Counting and Cardinality</b></p> <ul style="list-style-type: none"> <li>• Know number names and the count sequence.</li> <li>• Count to tell the number of objects.</li> <li>• Compare numbers.</li> </ul> <p><b>Operations and Algebraic Thinking</b></p> <ul style="list-style-type: none"> <li>• Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.</li> </ul> <p><b>Number and Operations in Base Ten</b></p> <ul style="list-style-type: none"> <li>• Work with numbers 11–19 to gain foundations for place value.</li> </ul>	<p><b>Measurement and Data</b></p> <ul style="list-style-type: none"> <li>• Describe and compare measurable attributes.</li> <li>• Classify objects and count the number of objects in categories.</li> </ul> <p><b>Geometry</b></p> <ul style="list-style-type: none"> <li>• Identify and describe shapes.</li> <li>• Analyze, compare, create, and compose shapes.</li> </ul>
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In **Grade 1**, instructional time should focus on four critical areas: (1) developing understanding of addition, subtraction, and strategies for addition and subtraction within 20; (2) developing understanding of whole number relationships and place value, including grouping in tens and ones; (3) developing understanding of linear measurement and measuring lengths as iterating length units; and (4) reasoning about attributes of, and composing and decomposing geometric shapes.

1. Students develop strategies for adding and subtracting whole numbers based on their prior work with small numbers. They use a variety of models, including discrete objects and length-based models (e.g., cubes connected to form lengths), to model add-to, take-from, put-together, take-apart, and compare situations to develop meaning for the operations of addition and subtraction, and to develop strategies to solve arithmetic problems with these operations. Students understand connections between counting and addition and subtraction (e.g., adding two is the same as counting on two). They use properties of addition to add whole numbers and to create and use increasingly sophisticated strategies based on these properties (e.g., “making tens”) to solve addition and subtraction problems within 20. By comparing a variety of solution strategies, children build their understanding of the relationship between addition and subtraction.
2. Students develop, discuss, and use efficient, accurate, and generalizable methods to add within 100 and subtract multiples of 10. They compare whole numbers (at least to 100) to develop understanding of and solve problems involving their relative sizes. They think of whole numbers between 10 and 100 in terms of tens and ones (especially recognizing the numbers 11 to 19 as composed of a ten and some ones). Through activities that build number sense, they understand the order of the counting numbers and their relative magnitudes.
3. Students develop an understanding of the meaning and processes of measurement, including underlying concepts such as iterating (the mental activity of building up the length of an object with equal-sized units) and the transitivity principle for indirect measurement.<sup>1</sup>
4. Students compose and decompose plane or solid figures (e.g., put two triangles together to make a quadrilateral) and build understanding of part-whole relationships as well as the properties of the original and composite shapes. As they combine shapes, they recognize them from different perspectives and orientations, describe their geometric attributes, and determine how they are alike and different, to develop the background for measurement and for initial understandings of properties such as congruence and symmetry.

<sup>1</sup> Students should apply the principle of transitivity of measurement to make indirect comparisons, but they need not use this technical term.

<p><b>Grade 1 Overview Operations and Algebraic Thinking</b></p> <ul style="list-style-type: none"> <li>• Represent and solve problems involving addition and subtraction.</li> <li>• Understand and apply properties of operations and the relationship between addition and subtraction.</li> <li>• Add and subtract within 20.</li> <li>• Work with addition and subtraction equations.</li> </ul> <p><b>Number and Operations in Base Ten</b></p> <ul style="list-style-type: none"> <li>• Extend the counting sequence.</li> <li>• Understand place value.</li> <li>• Use place value understanding and properties of operations to add and subtract.</li> </ul>	<p><b>Measurement and Data</b></p> <ul style="list-style-type: none"> <li>• Measure lengths indirectly and by iterating length units.</li> <li>• Tell and write time and money.</li> <li>• Represent and interpret data.</li> </ul> <p><b>Geometry</b></p> <ul style="list-style-type: none"> <li>• Reason with shapes and their attributes.</li> </ul>
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In **Grade 2**, instructional time should focus on four critical areas: (1) extending understanding of base-ten notation; (2) building fluency with addition and subtraction; (3) using standard units of measure; and (4) describing and analyzing shapes.

1. Students extend their understanding of the base-ten system. This includes ideas of counting in fives, tens, and multiples of hundreds, tens, and ones, as well as number relationships involving these units, including comparing. Students understand multi-digit numbers (up to 1000) written in base-ten notation, recognizing that the digits in each place represent amounts of thousands, hundreds, tens, or ones (e.g., 853 is 8 hundreds + 5 tens + 3 ones).
2. Students use their understanding of addition to develop fluency with addition and subtraction within 100. They solve problems within 1000 by applying their understanding of models for addition and subtraction, and they develop, discuss, and use efficient, accurate, and generalizable methods to compute sums and differences of whole numbers in base-ten notation, using their understanding of place value and the properties of operations. They select and accurately apply methods that are appropriate for the context and the numbers involved to mentally calculate sums and differences for numbers with only tens or only hundreds.
3. Students recognize the need for standard units of measure (centimeter and inch) and they use rulers and other measurement tools with the understanding that linear measure involves an iteration of units. They recognize that the smaller the unit, the more iterations they need to cover a given length.
4. Students describe and analyze shapes by examining their sides and angles. Students investigate, describe, and reason about decomposing and combining shapes to make other shapes. Through building, drawing, and analyzing two- and three-dimensional shapes, students develop a foundation for understanding area, volume, congruence, similarity, and symmetry in later grades.

<p><b>Grade 2 Overview Operations and Algebraic Thinking</b></p> <ul style="list-style-type: none"> <li>• Represent and solve problems involving addition and subtraction.</li> <li>• Add and subtract within 20.</li> <li>• Work with equal groups of objects to gain foundations for multiplication.</li> </ul> <p><b>Number and Operations in Base Ten</b></p> <ul style="list-style-type: none"> <li>• Understand place value.</li> <li>• Use place value understanding and properties of operations to add and subtract.</li> </ul>	<p><b>Measurement and Data</b></p> <ul style="list-style-type: none"> <li>• Measure and estimate lengths in standard units.</li> <li>• Relate addition and subtraction to length.</li> <li>• Work with time and money.</li> <li>• Represent and interpret data.</li> </ul> <p><b>Geometry</b></p> <ul style="list-style-type: none"> <li>• Reason with shapes and their attributes</li> </ul>
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In **Grade 3**, instructional time should focus on four critical areas: (1) developing understanding of multiplication and division and strategies for multiplication and division within 100; (2) developing understanding of fractions, especially unit fractions (fractions with numerator 1); (3) developing understanding of the structure of rectangular arrays and of area; and (4) describing and analyzing two-dimensional shapes.

1. Students develop an understanding of the meanings of multiplication and division of whole numbers through activities and problems involving equal-sized groups, arrays, and area models; multiplication is finding an unknown product, and division is finding an unknown factor in these situations. For equal-sized group situations, division can require finding the unknown number of groups or the unknown group size. Students use properties of operations to calculate products of whole numbers, using increasingly sophisticated strategies based on these properties to solve multiplication and division problems involving single-digit factors. By comparing a variety of solution strategies, students learn the relationship between multiplication and division.
2. Students develop an understanding of fractions, beginning with unit fractions. Students view fractions in general as being built out of unit fractions, and they use fractions along with visual fraction models to represent parts of a whole. Students understand that the size of a fractional part is relative to the size of the whole. For example,  $\frac{1}{2}$  of the paint in a small bucket could be less paint than  $\frac{1}{3}$  of the paint in a larger bucket, but  $\frac{1}{3}$  of a ribbon is longer than  $\frac{1}{5}$  of the same ribbon because when the ribbon is divided into 3 equal parts, the parts are longer than when the ribbon is divided into 5 equal parts. Students are able to use fractions to represent numbers equal to, less than, and greater than one. They solve problems that involve comparing fractions by using visual fraction models and strategies based on noticing equal numerators or denominators.
3. Students recognize area as an attribute of two-dimensional regions. They measure the area of a shape by finding the total number of same-size units of area required to cover the shape without gaps or overlaps, a square with sides of unit length being the standard unit for measuring area. Students understand that rectangular arrays can be decomposed into identical rows or into identical columns. By decomposing rectangles into rectangular arrays of squares, students connect area to multiplication, and justify using multiplication to determine the area of a rectangle.
4. Students describe, analyze, and compare properties of two-dimensional shapes. They compare and classify shapes by their sides and angles, and connect these with definitions of shapes. Students also relate their fraction work to geometry by expressing the area of part of a shape as a unit fraction of the whole.

<p><b>Grade 3 Overview Operations and Algebraic Thinking</b></p> <ul style="list-style-type: none"> <li>• Represent and solve problems involving multiplication and division.</li> <li>• Understand properties of multiplication and the relationship between multiplication and division.</li> <li>• Multiply and divide within 100.</li> <li>• Solve problems involving the four operations, and identify and explain patterns in arithmetic.</li> </ul> <p><b>Number and Operations in Base Ten</b></p> <ul style="list-style-type: none"> <li>• Use place value understanding and properties of operations to perform multi-digit arithmetic.</li> </ul> <p><b>Number and Operations—Fractions</b></p> <ul style="list-style-type: none"> <li>• Develop understanding of fractions as numbers.</li> </ul>	<p><b>Measurement and Data</b></p> <ul style="list-style-type: none"> <li>• Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.</li> <li>• Represent and interpret data.</li> <li>• Geometric measurement: understand concepts of area and relate area to multiplication and to addition.</li> <li>• Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.</li> </ul> <p><b>Geometry</b></p> <ul style="list-style-type: none"> <li>• Reason with shapes and their attributes</li> </ul>
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In **Grade 4**, instructional time should focus on three critical areas: (1) developing understanding and fluency with multi-digit multiplication, and developing understanding of dividing to find quotients involving multi-digit dividends; (2) developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers; (3) understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, particular angle measures, and symmetry.

1. Students generalize their understanding of place value to 1,000,000, understanding the relative sizes of numbers in each place. They apply their understanding of models for multiplication (equal-sized groups, arrays, area models), place value, and properties of operations, in particular the distributive property, as they develop, discuss, and use efficient, accurate, and generalizable methods to compute products of multi-digit whole numbers. Depending on the numbers and the context, they select and accurately apply appropriate methods to estimate or mentally calculate products. They develop fluency with efficient procedures for multiplying whole numbers; understand and explain why the procedures work based on place value and properties of operations; and use them to solve problems. Students apply their understanding of models for division, place value, properties of operations, and the relationship of division to multiplication as they develop, discuss, and use efficient, accurate, and generalizable procedures to find quotients involving multi-digit dividends. They select and accurately apply appropriate methods to estimate and mentally calculate quotients, and interpret remainders based upon the context.
2. Students develop understanding of fraction equivalence and operations with fractions. They recognize that two different fractions can be equal (e.g.,  $15/9 = 5/3$ ), and they develop methods for generating and recognizing equivalent fractions. Students extend previous understandings about how fractions are built from unit fractions, composing fractions from unit fractions, decomposing fractions into unit fractions, and using the meaning of fractions and the meaning of multiplication to multiply a fraction by a whole number.
3. Students describe, analyze, compare, and classify two-dimensional shapes. Through building, drawing, and analyzing two-dimensional shapes, students deepen their understanding of properties of two-dimensional objects and the use of them to solve problems involving symmetry.

<p><b>Grade 4 Overview Operations and Algebraic Thinking</b></p> <ul style="list-style-type: none"> <li>• Use the four operations with whole numbers to solve problems.</li> <li>• Gain familiarity with factors and multiples.</li> <li>• Generate and analyze patterns.</li> </ul> <p><b>Number and Operations in Base Ten</b></p> <ul style="list-style-type: none"> <li>• Generalize place value understanding for multidigit whole numbers.</li> <li>• Use place value understanding and properties of operations to perform multi-digit arithmetic.</li> </ul> <p><b>Number and Operations—Fractions</b></p> <ul style="list-style-type: none"> <li>• Extend understanding of fraction equivalence and ordering.</li> <li>• Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.</li> <li>• Understand decimal notation for fractions, and compare decimal fractions.</li> </ul>	<p><b>Measurement and Data</b></p> <ul style="list-style-type: none"> <li>• Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.</li> <li>• Represent and interpret data.</li> <li>• Geometric measurement: understand concepts of angle and measure angles.</li> </ul> <p><b>Geometry</b></p> <ul style="list-style-type: none"> <li>• Draw and identify lines and angles, and classify shapes by properties of their lines and angles.</li> </ul>
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In **Grade 5**, instructional time should focus on three critical areas: (1) developing fluency with addition and subtraction of fractions, and developing understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions); (2) extending division to 2-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations; and (3) developing understanding of volume.

1. Students apply their understanding of fractions and fraction models to represent the addition and subtraction of fractions with unlike denominators as equivalent calculations with like denominators. They develop fluency in calculating sums and differences of fractions, and make reasonable estimates of them. Students also use the meaning of fractions, of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for multiplying and dividing fractions make sense. (Note: this is limited to the case of dividing unit fractions by whole numbers and whole numbers by unit fractions.)
2. Students develop understanding of why division procedures work based on the meaning of base-ten numerals and properties of operations. They finalize fluency with multi-digit addition, subtraction, multiplication, and division. They apply their understandings of models for decimals, decimal notation, and properties of operations to add and subtract decimals to hundredths. They develop fluency in these computations, and make reasonable estimates of their results. Students use the relationship between decimals and fractions, as well as the relationship between finite decimals and whole numbers (i.e., a finite decimal multiplied by an appropriate power of 10 is a whole number), to understand and explain why the procedures for multiplying and dividing finite decimals make sense. They compute products and quotients of decimals to hundredths efficiently and accurately.
3. Students recognize volume as an attribute of three-dimensional space. They understand that volume can be measured by finding the total number of same-size units of volume required to fill the space without gaps or overlaps. They understand that a 1-unit by 1-unit by 1-unit cube is the standard unit for measuring volume. They select appropriate units, strategies, and tools for solving problems that involve estimating and measuring volume. They decompose three-dimensional shapes and find volumes of right rectangular prisms by viewing them as decomposed into layers of arrays of cubes. They measure necessary attributes of shapes in order to determine volumes to solve real world and mathematical problems.

<p><b>Grade 5 Overview Operations and Algebraic Thinking</b></p> <ul style="list-style-type: none"> <li>• Write and interpret numerical expressions.</li> <li>• Analyze patterns and relationships.</li> </ul> <p><b>Number and Operations in Base Ten</b></p> <ul style="list-style-type: none"> <li>• Understand the place value system.</li> <li>• Perform operations with multi-digit whole numbers and with decimals to hundredths.</li> </ul> <p><b>Number and Operations—Fractions</b></p> <ul style="list-style-type: none"> <li>• Use equivalent fractions as a strategy to add and subtract fractions.</li> <li>• Apply and extend previous understandings of multiplication and division to multiply and divide fractions.</li> </ul>	<p><b>Measurement and Data</b></p> <ul style="list-style-type: none"> <li>• Convert like measurement units within a given measurement system.</li> <li>• Represent and interpret data.</li> <li>• Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.</li> </ul> <p><b>Geometry</b></p> <ul style="list-style-type: none"> <li>• Graph points on the coordinate plane to solve real-world and mathematical problems.</li> <li>• Classify two-dimensional figures into categories based on their properties.</li> </ul>
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In **Grade 6**, instructional time should focus on four critical areas: (1) connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems; (2) completing understanding of division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers; (3) writing, interpreting, and using expressions and equations; and (4) developing understanding of statistical thinking.

1. Students use reasoning about multiplication and division to solve ratio and rate problems about quantities. By viewing equivalent ratios and rates as deriving from, and extending, pairs of rows (or columns) in the multiplication table, and by analyzing simple drawings that indicate the relative size of quantities, students connect their understanding of multiplication and division with ratios and rates. Thus students expand the scope of problems for which they can use multiplication and division to solve problems, and they connect ratios and fractions. Students solve a wide variety of problems involving ratios and rates.
2. Students use the meaning of fractions, the meanings of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for dividing fractions make sense. Students use these operations to solve problems. Students extend their previous understandings of number and the ordering of numbers to the full system of rational numbers, which includes negative rational numbers, and in particular negative integers. They reason about the order and absolute value of rational numbers and about the location of points in all four quadrants of the coordinate plane.
3. Students understand the use of variables in mathematical expressions. They write expressions and equations that correspond to given situations, evaluate expressions, and use expressions and formulas to solve problems. Students understand that expressions in different forms can be equivalent, and they use the properties of operations to rewrite expressions in equivalent forms. Students know that the solutions of an equation are the values of the variables that make the equation true. Students use properties of operations and the idea of maintaining the equality of both sides of an equation to solve simple one-step equations. Students construct and analyze tables, such as tables of quantities that are in equivalent ratios, and they use equations (such as  $3x = y$ ) to describe relationships between quantities.
4. Building on and reinforcing their understanding of number, students begin to develop their ability to think statistically. Students recognize that a data distribution may not have a definite center and that different ways to measure center yield different values. The median measures center in the sense that it is roughly the middle value. The mean measures center in the sense that it is the value that each data point would take on if the total of the data values were redistributed equally, and also in the sense that it is a balance point. Students recognize that a measure of variability (interquartile range or mean absolute deviation) can also be useful for summarizing data because two very different sets of data can have the same mean and median yet be distinguished by their variability. Students learn to describe and summarize numerical data sets, identifying clusters, peaks, gaps, and symmetry, considering the context in which the data were collected.

<p><b>Grade 6 Overview Ratios and Proportional Relationships</b></p> <ul style="list-style-type: none"> <li>• Understand ratio concepts and use ratio reasoning to solve problems.</li> </ul> <p><b>The Number System</b></p> <ul style="list-style-type: none"> <li>• Apply and extend previous understandings of multiplication and division to divide fractions by fractions.</li> <li>• Compute fluently with multi-digit numbers and find common factors and multiples.</li> <li>• Apply and extend previous understandings of numbers to the system of rational numbers.</li> </ul>	<p><b>Expressions and Equations</b></p> <ul style="list-style-type: none"> <li>• Apply and extend previous understandings of arithmetic to algebraic expressions.</li> <li>• Reason about and solve one-variable equations and inequalities.</li> <li>• Represent and analyze quantitative relationships between dependent and independent variables.</li> </ul> <p><b>Geometry</b></p> <ul style="list-style-type: none"> <li>• Solve real-world and mathematical problems involving area, surface area, and volume.</li> </ul> <p><b>Statistics and Probability</b></p> <ul style="list-style-type: none"> <li>• Develop understanding of statistical variability.</li> <li>• Summarize and describe distributions</li> </ul>
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In **Grade 7**, instructional time should focus on four critical areas: (1) developing understanding of and applying proportional relationships; (2) developing understanding of operations with rational numbers and working with expressions and linear equations; (3) solving problems involving scale drawings and informal geometric constructions, and working with two- and three-dimensional shapes to solve problems involving area, surface area, and volume; and (4) drawing inferences about populations based on samples.

1. Students extend their understanding of ratios and develop understanding of proportionality to solve single- and multi-step problems. Students use their understanding of ratios and proportionality to solve a wide variety of percent problems, including those involving discounts, interest, taxes, tips, and percent increase or decrease. Students solve problems about scale drawings by relating corresponding lengths between the objects or by using the fact that relationships of lengths within an object are preserved in similar objects. Students graph proportional relationships and understand the unit rate informally as a measure of the steepness of the related line, called the slope. They distinguish proportional relationships from other relationships.
2. Students develop a unified understanding of number, recognizing fractions, decimals (that have a finite or a repeating decimal representation), and percents as different representations of rational numbers. Students extend addition, subtraction, multiplication, and division to all rational numbers, maintaining the properties of operations and the relationships between addition and subtraction, and multiplication and division. By applying these properties, and by viewing negative numbers in terms of everyday contexts (e.g., amounts owed or temperatures below zero), students explain and interpret the rules for adding, subtracting, multiplying, and dividing with negative numbers. They use the arithmetic of rational numbers as they formulate expressions and equations in one variable and use these equations to solve problems.
3. Students continue their work with area from Grade 6, solving problems involving the area and circumference of a circle and surface area of three-dimensional objects. In preparation for work on congruence and similarity in Grade 8 they reason about relationships among two-dimensional figures using scale drawings and informal geometric constructions, and they gain familiarity with the relationships between angles formed by intersecting lines. Students work with three-dimensional figures, relating them to two-dimensional figures by examining cross-sections. They solve real-world and mathematical problems involving area, surface area, and volume of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes and right prisms.
4. Students build on their previous work with single data distributions to compare two data distributions and address questions about differences between populations. They begin informal work with random sampling to generate data sets and learn about the importance of representative samples for drawing inferences.

<p><b>Grade 7 Overview Ratios and Proportional Relationships</b></p> <ul style="list-style-type: none"> <li>• Analyze proportional relationships and use them to solve real-world and mathematical problems.</li> </ul> <p><b>The Number System</b></p> <ul style="list-style-type: none"> <li>• Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.</li> </ul> <p><b>Expressions and Equations</b></p> <ul style="list-style-type: none"> <li>• Use properties of operations to generate equivalent expressions.</li> <li>• Solve real-life and mathematical problems using numerical and algebraic expressions and equations.</li> </ul>	<p><b>Geometry</b></p> <ul style="list-style-type: none"> <li>• Draw, construct and describe geometrical figures and describe the relationships between them.</li> <li>• Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.</li> </ul> <p><b>Statistics and Probability</b></p> <ul style="list-style-type: none"> <li>• Use random sampling to draw inferences about a population.</li> <li>• Draw informal comparative inferences about two populations.</li> <li>• Investigate chance processes and develop, use, and evaluate probability models.</li> </ul>
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In **Grade 8**, instructional time should focus on three critical areas: (1) formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations; (2) grasping the concept of a function and using functions to describe quantitative relationships; (3) analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem.

1. Students use linear equations and systems of linear equations to represent, analyze, and solve a variety of problems. Students recognize equations for proportions ( $y/x = m$  or  $y = mx$ ) as special linear equations ( $y = mx + b$ ), understanding that the constant of proportionality ( $m$ ) is the slope, and the graphs are lines through the origin. They understand that the slope ( $m$ ) of a line is a constant rate of change, so that if the input or  $x$ -coordinate changes by an amount  $A$ , the output or  $y$ -coordinate changes by the amount  $m \cdot A$ . Students also use a linear equation to describe the association between two quantities in bivariate data (such as arm span vs. height for students in a classroom). At this grade, fitting the model, and assessing its fit to the data are done informally. Interpreting the model in the context of the data requires students to express a relationship between the two quantities in question and to interpret components of the relationship (such as slope and  $y$ -intercept) in terms of the situation. Students strategically choose and efficiently implement procedures to solve linear equations in one variable, understanding that when they use the properties of equality and the concept of logical equivalence, they maintain the solutions of the original equation. Students solve systems of two linear equations in two variables and relate the systems to pairs of lines in the plane; these intersect, are parallel, or are the same line. Students use linear equations, systems of linear equations, linear functions, and their understanding of slope of a line to analyze situations and solve problems.
2. Students grasp the concept of a function as a rule that assigns to each input exactly one output. They understand that functions describe situations where one quantity determines another. They can translate among representations and partial representations of functions (noting that tabular and graphical representations may be partial representations), and they describe how aspects of the function are reflected in the different representations.
3. Students use ideas about distance and angles, how they behave under translations, rotations, reflections, and dilations, and ideas about congruence and similarity to describe and analyze two-dimensional figures and to solve problems. Students show that the sum of the angles in a triangle is the angle formed by a straight line, and that various configurations of lines give rise to similar triangles because of the angles created when a transversal cuts parallel lines. Students understand the statement of the Pythagorean Theorem and its converse, and can explain why the Pythagorean Theorem holds, for example, by decomposing a square in two different ways. They apply the Pythagorean Theorem to find distances between points on the coordinate plane, to find lengths, and to analyze polygons. Students complete their work on volume by solving problems involving cones, cylinders, and spheres.

<p><b>Grade 8 Overview The Number System</b></p> <ul style="list-style-type: none"> <li>• Know that there are numbers that are not rational, and approximate them by rational numbers.</li> </ul> <p><b>Expressions and Equations</b></p> <ul style="list-style-type: none"> <li>• Work with radicals and integer exponents.</li> <li>• Understand the connections between proportional relationships, lines, and linear equations.</li> <li>• Analyze and solve linear equations and pairs of simultaneous linear equations.</li> </ul> <p><b>Functions</b></p> <ul style="list-style-type: none"> <li>• Define, evaluate, and compare functions.</li> <li>• Use functions to model relationships between quantities.</li> </ul>	<p><b>Geometry</b></p> <ul style="list-style-type: none"> <li>• Understand congruence and similarity using physical models, transparencies, or geometry software.</li> <li>• Understand and apply the Pythagorean Theorem.</li> <li>• Solve real-world and mathematical problems involving volume of cylinders, cones and spheres.</li> </ul> <p><b>Statistics and Probability</b></p> <ul style="list-style-type: none"> <li>• Investigate patterns of association in bivariate data.</li> </ul>
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