Lesson 2: Estimating Probabilities by Collecting Data

Classwork

Example 1: Carnival Game

At the school carnival, there is a game in which students spin a large spinner. The spinner has four equal sections numbered 1–4 as shown below. To play the game, a student spins the spinner twice and adds the two numbers that the spinner lands on. If the sum is greater than or equal to 5, the student wins a prize.
Exercises 1–8
You and your partner will play this game 15 times. Record the outcome of each spin in the table below.

<table>
<thead>
<tr>
<th>Turn</th>
<th>1st Spin Results</th>
<th>2nd Spin Results</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>4</td>
<td>1 + 4 = 5</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>4</td>
<td>4 + 4 = 8</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>1</td>
<td>1 + 1 = 2</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>4</td>
<td>3 + 4 = 7</td>
</tr>
<tr>
<td>5</td>
<td>4</td>
<td>3</td>
<td>4 + 3 = 7</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
<td>4</td>
<td>2 + 4 = 6</td>
</tr>
<tr>
<td>7</td>
<td>4</td>
<td>1</td>
<td>4 + 1 = 5</td>
</tr>
<tr>
<td>8</td>
<td>3</td>
<td>1</td>
<td>3 + 1 = 4</td>
</tr>
<tr>
<td>9</td>
<td>1</td>
<td>4</td>
<td>1 + 4 = 5</td>
</tr>
<tr>
<td>10</td>
<td>1</td>
<td>1</td>
<td>1 + 1 = 2</td>
</tr>
<tr>
<td>11</td>
<td>4</td>
<td>2</td>
<td>4 + 2 = 6</td>
</tr>
<tr>
<td>12</td>
<td>3</td>
<td>3</td>
<td>3 + 3 = 6</td>
</tr>
<tr>
<td>13</td>
<td>1</td>
<td>4</td>
<td>1 + 4 = 5</td>
</tr>
<tr>
<td>14</td>
<td>1</td>
<td>3</td>
<td>1 + 3 = 4</td>
</tr>
<tr>
<td>15</td>
<td>2</td>
<td>2</td>
<td>2 + 2 = 4</td>
</tr>
</tbody>
</table>
1. Out of the 15 turns how many times was the sum greater than or equal to 5?

10 times

2. What sum occurred most often?

5

3. What sum occurred least often?

3 didn’t occur at all

4. If students played a lot of games, what proportion of the games played will they win? Explain your answer.

\[
\frac{10}{15} = \frac{2}{3} = .66\overline{6} = 66.6\%\]

5. Name a sum that would be impossible to get while playing the game.

0, 1 or any # greater than 8

6. What event is certain to occur while playing the game?

2, 3, 4, 5, 6, 7, 8
When you were spinning the spinner and recording the outcomes, you were performing a chance experiment. You can use the results from a chance experiment to estimate the probability of an event. In the example above, you spun the spinner 15 times and counted how many times the sum was greater than or equal to 5. An estimate for the probability of a sum greater than or equal to 5 is:

\[ p(\text{sum } \geq 5) = \frac{\text{Number of observed occurrences of the event}}{\text{Total number of observations}} \]

7. Based on your experiment of playing the game, what is your estimate for the probability of getting a sum of 5 or more?

\[ \frac{10}{15} \div 5 = \frac{2}{3} = \frac{10}{15} = 66 \frac{2}{3}\% \]

8. Based on your experiment of playing the game, what is your estimate for the probability of getting a sum of exactly 5?

\[ \frac{4}{15} = 26 \approx 27\% \]
Example 2: Animal Crackers

A student brought a very large jar of animal crackers to share with students in class. Rather than count and sort all the different types of crackers, the student randomly chose 20 crackers and found the following counts for the different types of animal crackers:

<table>
<thead>
<tr>
<th>Type</th>
<th>Count</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lion</td>
<td>2</td>
</tr>
<tr>
<td>Camel</td>
<td>1</td>
</tr>
<tr>
<td>Monkey</td>
<td>4</td>
</tr>
<tr>
<td>Elephant</td>
<td>5</td>
</tr>
<tr>
<td>Zebra</td>
<td>3</td>
</tr>
<tr>
<td>Penguin</td>
<td>3</td>
</tr>
<tr>
<td>Tortoise</td>
<td>2</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>20</strong></td>
</tr>
</tbody>
</table>

The student can now use that data to find an estimate for the probability of choosing a zebra from the jar by dividing the observed number of zebras by the total number of crackers selected. The estimated probability of picking a zebra is \( \frac{3}{20} \) or 0.15 or 15%. This means that an estimate of the proportion of the time a zebra will be selected is 0.15, or 15% of the time. This could be written as \( P(zebra) = 0.15 \), or the probability of selecting a zebra is 0.15.
Exercises 9–15

If a student were to randomly select a cracker from the large jar:

9. What is your estimate for the probability of selecting a lion?

\[ P(\text{lion}) = \frac{2}{20} = \frac{1}{10} = 0.1 = 10\% \]

10. What is the estimate for the probability of selecting a monkey?

\[ P(\text{monkey}) = \frac{4}{20} = \frac{1}{5} = 0.2 = 20\% \]

11. What is the estimate for the probability of selecting a penguin or a camel?

\[ P(\text{penguin or camel}) = \frac{3+1}{20} = \frac{4}{20} = \frac{1}{5} = 0.2 = 20\% \]
12. What is the estimate for the probability of selecting a rabbit?

\[ P(\text{rabbit}) = \frac{0}{20} = 0 = 0\% \]

13. Is there the same number of each animal cracker in the large jar? Explain your answer.

No, there appears to be more elephants than other types of crackers.

14. If the student were to randomly select another 20 animal crackers, would the same results occur? Why or why not?

Probably not. Results may be similar, but it is unlikely they would be exactly the same.

15. If there are 500 animal crackers in the jar, how many elephants are in the jar? Explain your answer.

\[ P(\text{elephant}) = \frac{5}{20} \div = \frac{1}{4} = 0.25 \]

\[ 0.25 \times 500 = 125 \]

The estimate for the number of elephants would be 125.