Examples of Key Advances in Kindergarten

- Students orally count to 100, beginning from any given number within 100, to support their later ability to count higher, as well as to develop a pattern of tens as they become skilled with naming the next ten (e.g. "forty-nine, fifty").
- Students pair objects 1:1 with counting words, and they learn that the last number word tells the total number of objects in a collection (up to 20). This is called "cardinal counting," as opposed to "rote counting" (reciting the counting words in order).
- Students use their ability to subitize (recognize small quantities at a glance) to help them compose and decompose numbers. For example, when students are using objects to show the decompositions 5 = 2 + 3 or 5 = 4 + 1, it is helpful for them to be able to subitize two or three objects.
- Students anchor to 5, realizing that 6 is one more than 5 and 4 is one less.
- Students build the crucial basis for place-value understanding of teen numbers by learning to anchor to 10 and to compose or decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (e.g., 18 = 10 + 8). This is also a crucial prerequisite for the grade 1 adding-and-subtracting strategy of making a ten and for the meaningful learning of writing numbers from 1 to 20.
- Students compare the number of objects in one group versus the number of objects in another group to find which has more or less, and eventually compare written numerals 1–10 to find which number describes more or less than another number.
- Students understand addition as joining collections and adding to collections, and they understand subtraction as taking collections apart or taking from collections. They represent these operations in a variety of ways.

Fluency Expectations or Examples of Culminating Standards

K.CC.A.1 Count to 100 by tens and ones.

K.CC.A.3 Write numbers from 0 to 20. Represent a number of objects with a written numeral 0–20 (with 0 representing a count of no objects).

It is recommended that, throughout the year, students work toward fluency in writing the numerals 0–10. Note that learning to write numerals is generally more difficult than

learning to read them. It is common for students to reverse numbers at this stage (e.g., writing Ɛ for 3).[11]

K.CC.B.5 Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects.

It is recommended that students become fluent in cardinal counting with sets ≤ 5 ***early in the year*** and with increasingly large sets as the year progresses.

K.CC.C.7 Compare two numbers between 1 and 10 presented as written numerals.

If students are less than fluent in number comparisons by the end of kindergarten, then they may not have mastered early number concepts. Note that K.CC.C.6 (Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies) is a precursor to K.CC.C.7 and portrays the kind of concrete work that students should be doing en route to mastering numeral-based comparisons.

K.OA.A.5 Fluently add and subtract within 5.

Given an oral or written expression with any two numbers 0–5 with a sum less than or equal to 5 (e.g., "three and one" or 3 + 1), students can find the sum reasonably quickly, and say or write it. For subtractions involving numbers of the same sizes, and given an oral or written expression (e.g., "four, take away one" or 4 – 1), students can find the difference reasonably quickly and say or write it. Some students may still need to use fingers or make drawings. Students grow in fluency throughout the year as they work with addition and subtraction situations.

Examples of Major Within-Grade Dependencies

- Much of the learning in kindergarten—K.CC.C.6, all of K.OA and K.NBT, and K.MD.B.3—depends on the foundational ability to count to answer "how many?" (K.CC.B.5), which itself is grounded in K.CC.B.4.

Examples of Opportunities for Connections among Standards, Clusters, or Domains

- In addition to laying the groundwork for place value in grade 1, working with numbers 11–19 (K.NBT.A.1) provides opportunities for cardinal counting beyond 10 (see K.CC.B.5) and for writing two-digit numbers (see K.CC.A.3). Ten frames, strips with ten ones and some loose ones, and math drawings can be helpful for this work.

[11] Material adapted from National Research Council. *Mathematics Learning in Early Childhood: Paths Toward Excellence and Equity* (Washington, DC: The National Academies Press, 2009), p. 138.

- K.MD.B.3 provides opportunities for cardinal counting (see K.CC.B.5) and for comparing numbers (see K.CC.C.6). K.MD.B.3 also offers a context in which to decompose 10 in more than one way (see K.OA.A.3).
- K.G.A.2 and K.G.B.4 offer some opportunities for counting and comparing numbers.

Examples of Opportunities for In-Depth Focus

K.CC.B.5 Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects.

Cardinal counting is itself a needed focus and is a main component of other work in the kindergarten classroom. Opportunities to develop students' understanding of cardinality abound, both within the instructional time devoted specifically to mathematics and elsewhere in the instructional day—for example, "How many plants did the class plant for the science project? What if we planted one more?"

K.OA.A.2 Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.

Through representing and solving addition and subtraction problems, students understand addition as joining and adding to and understand subtraction as separating and taking from. Initially, the meaning of addition is separate from the meaning of subtraction, and students build relationships between addition and subtraction over time, with subtraction coming to be understood as reversing the actions involved in addition and as finding an unknown addend.[12]

K.OA.A.3 Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).

The ability to decompose numbers flexibly is a key focus, and is connected with other standards, such as K.OA.A.1 and K.OA.A.2. Decomposing numbers is an important foundation for adding and subtracting numbers below ten and for beginning to understand relationships between adding and subtracting. Knowing decompositions is also one of the three prerequisites for using the grade 1 make-a-ten strategy to add numbers with a teen total (see K.OA.A.4 below).

K.OA.A.4 For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.

[12] See pages 2 and 3 of the *K–5 Progressions Document for Counting and Cardinality and Operations and Algebraic Thinking*, available at http://math.arizona.edu/~ime/progressions/ (2014).

In grade 1, "making ten" will become a key strategy for adding and subtracting within 20; students gain the foundations for the first step of this strategy in kindergarten by finding the number that makes 10 when given another number. Over the course of the year, given frequent opportunities (e.g., a "how many fingers don't you see" game), many kindergarten children can become fluent with the pairs of numbers that make 10 and can, when a number less than 10 is named, name the "missing amount" even without looking at fingers. Showing numbers from 6 to 10 in groups of 5 (e.g., a top row of 5 circles with 1 to 5 circles below it) is a helpful visual support for finding the pairs of numbers that make ten.

K.NBT.1 Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (such as 18 = 10 + 8); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.

Understanding each number 11–19 as a collection of ten and some ones (e.g., 16 = 10 + 6) requires extensive experience counting out and separating ten and the ones and relating these groups to the written numerals. Place-value cards in which the ones numeral can be placed over the 0 in the 10 (the ten card is twice as wide as the ones cards) can be helpful in connecting to the concepts in K.NBT.A.1, K.CC.B.5, and K.CC.A.3.

Examples of Opportunities for Connecting Mathematical Content and Mathematical Practices

Mathematical practices should be evident throughout mathematics instruction and should be centrally connected to the most important work of the grade. These mathematical practices can be summarized in the following sentence: Teachers help students to do mathematical sense-making about mathematical structure using mathematical drawings (or objects) to support mathematical explaining (MP 1 & 6; 7 & 8; 4 & 5; 2 & 3). Some brief examples of how the content of this grade might be connected to the Standards for Mathematical Practice follow.

- Kindergarten students say the number names by ones and by tens ("ten, twenty, thirty, . . .") all the way to 100 (K.CC.A.1). The structure of a number name such as "thirty-two" reflects the underlying system of place value. Attending to and using that structure (MP.7) is an important foundation for place value. See the K–5 progressions document for Counting and Cardinality and Operations and Algebraic Thinking for more information about how patterns in the number names affect learning the teen numbers (including deviations from those patterns, as in "sixteen," in which the digit in the ones place is said first but written second).
- As students count by tens (K.CC.A.1), they may make sense (MP.1) of these numbers by reciting each new number in the sequence "ten, twenty, thirty . . ." as a new child joins the children already standing in front of the classroom and showing all their fingers. The patterns in the place-value system—the structure of numbers (MP.7)—become more apparent when children say "six tens is sixty, seven tens is seventy, eight tens is eighty," etc. Children can also flash ten fingers as they count by tens to feel the ten that are added on with each count.

- When students progress from drawing realistic (artistic) pictures of situations to diagramming addition and subtraction situations using circles or other symbols, and making connections between them, they are relating the concrete to the abstract (MP.2) and making their first mathematical models (MP.4). Equations to describe these situations (such as 8 + 2 = 10) are also mathematical models.
- A student choosing to use objects, fingers, or a math drawing to represent and solve a word problem is an example of the student using an appropriate tool strategically (MP.5).

Manipulatives such as physical models of hundreds, tens, and ones, and visual models such as math drawings, are important parts of the K–2 classroom. These manipulatives and visual models should always be connected to written symbols and methods.

A note on manipulatives in grades K–2: Manipulatives such as physical models of hundreds, tens, and ones, and visual models such as math drawings, are important parts of the K–2 classroom. These manipulatives and visual models should always be connected to written symbols and methods.[13]

Content Emphases by Cluster

Not all of the content in a given grade is emphasized equally in the standards. Some clusters require greater emphasis than others, based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness. In addition, an intense focus on the most critical material at each grade allows depth in learning, which is carried out through the Standards for Mathematical Practice.

To say that some topics have greater emphasis is not to say that any content in the standards can safely be neglected in instruction. Neglecting material will leave gaps in student skill and understanding and may leave students unprepared for the challenges of a later grade. All standards figure in a mathematical education and will therefore be eligible for inclusion on PARCC assessments. However, the assessments will strongly focus where the standards strongly focus.

The following table identifies the Major Clusters, Supporting Clusters, and Additional Clusters for kindergarten; below the table, suggestions are given for ways to connect the Supporting Clusters to the Major Clusters of the grade. Thus, rather than suggesting, even inadvertently, that some material should not be taught, this document provides direct advice for teaching grade-level material in ways that foster greater focus and coherence.

[13] See page 19 of *K–8 Publishers' Criteria for the Common Core State Standards for Mathematics*, available at http://www.corestandards.org/assets/Math_Publishers_Criteria_K-8_Summer%202012_FINAL.pdf. (2014)

Key: Major Clusters; Supporting Clusters; Additional Clusters

Counting and Cardinality

- Know number names and the count sequence.
- Count to tell the number of objects.
- Compare numbers.

Operations and Algebraic Thinking

- Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

Number and Operations in Base Ten

- Work with numbers 11–19 to gain foundations for place value.

Measurement and Data

- Describe and compare measurable attributes.
- Classify objects and count the number of objects in categories.

Geometry

- Identify and describe shapes.
- Analyze, compare, create, and compose shapes.

Examples of Linking Supporting Clusters to the Major Work of the Grade

So much is brand new to children in kindergarten that, as much as possible, everything throughout the school day should support everything else, as, for example, when language supports number.

- Even within mathematics itself, understanding, for example, that 18 is ten ones and eight more ones (K.NBT.A.1) requires, but also supports, understanding what it means to combine 10 and 8 or to take apart 18 (K.OA).
- K.MD.B.3 offers a context in which to decompose numbers less than or equal to 10 in more than one way (see K.OA.A.3).

Examples of Key Advances from Kindergarten to Grade 1

- Students gradually come to employ mental strategies (such as counting on and making ten) that make use of embedded concepts of number and the properties of addition and subtraction; by contrast, kindergarten students determine sums and differences primarily by representing problems with objects or drawings.
- Students read and write numbers through 120 and learn the early elements of place value, in particular being able to think of a ten as a unit and understanding that the digits of a two-digit number represent the number of tens in that number and the number of remaining ones.
- Students use their understanding of place value and the properties of operations to represent, explain, and perform addition and subtraction of two-digit numbers in specified cases.
- Students represent and solve a large variety of addition and subtraction problems—that is, word problems, and problems set in classroom discussions, that involve addition and subtraction situations such as adding to, taking from, putting together, taking apart, comparing, etc., with different unknown quantities in the problem.[14]
- Students write equations for a variety of reasons, such as expressing a decomposition of a number (16 = 9 + 7), expressing a piece of reasoning about numbers (9 + 7 = 9 + 1 + 6, along the way to making ten), or representing a word problem with an unknown (9 + ? = 16). Students use the equal sign appropriately, evaluate the truth of an equation, and determine unknown numbers that will make an equation true. Students make connections among concrete objects, pictorial representations, and equations.

Fluency Expectations or Examples of Culminating Standards

1.OA.C.6 Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).

Fluency is demonstrated as that, given any two numbers 0–10 with a sum less than or equal to 20, students can say the sum reasonably quickly, and likewise, for related differences, given one number and a total that is 10 or less, they can reasonably quickly say the amount taken away or the unknown addend. Students grow in fluency throughout the year as they work with addition and subtraction situations.

[14] See Table 2 on page 9 of the *K–5 Progressions Document for Counting and Cardinality and Operations and Algebraic Thinking*, available at http://math.arizona.edu/~ime/progressions/ (2014). This table is based on Table 1 on page 88 of the CCSS for Mathematics.

1.OA.D.7 Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. For example, which of the following equations are true, and which are false? 6 = 6, 7 = 8 – 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2.

This standard relates to fluency when the additions and subtractions in the equations fall within 10, as they do in the italicized examples accompanying the standard.

1.OA.D.8 Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations 8 + ? = 11, 5 = ☐ – 3, 6 + 6 = ☐.

A crucial aspect of understanding and solving such equations is knowing where the total is in addition equations (alone on one side) and in subtraction equations (before the minus sign). Also important is that students see varied equation forms, especially those with only one number on the left side of the equation.

1.NBT.C.5 Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.

Quickly finding 10 more or 10 less than a two-digit number is best thought of as an indicator of whether students have an understanding of place value for two-digit numbers.

Examples of Major Within-Grade Dependencies

- 1.OA.B.3 calls for students to "apply properties of operations" and gives the example "If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known." Similarly, knowing 13 – 3 gives a good starting place for figuring out 13 – 4. Use of properties lets students apply knowledge that they have to situations that they need to figure out.
- 1.NBT.B.2 describes the place-value foundations for 1.NBT.B.3 and 1.NBT.C.4. Comparing numbers (1.NBT.B.3) involves thinking about the sizes of tens and ones, and adding two-digit numbers (1.NBT.C.4) involves adding tens with tens and ones with ones, and sometimes composing a ten. These ideas and methods rest on an understanding of the place-value units and the use of visual models of these units in solving and explaining problems using these standards.

Examples of Opportunities for Connections among Standards, Clusters, or Domains

- A thorough understanding of how place-value language and notation represent number (cluster 1.NBT.A) is needed for meaningful calculation (cluster 1.NBT.B) in many ways—not just pencil-and-paper calculation, but mental calculation as well. For purposes of calculation, it is valuable to use the tens and ones in two-digit numbers, single-digit knowledge, and properties of the operations (1.OA.B.4). In Grade 1, calculation ranges from simple mental adding, such as 40 + 20 (add the 4 tens and 2 tens) and 58 + 6 (6 gives 2 to 58 to make 60, then 60 plus the 4 left in 6 equals 64), to the more complex cases that require composing ten ones to make a ten, such as 37 + 56.

- The study of word problems in grade 1 (1.OA.A.1, 1.OA.A.2) can be coordinated with students' growing proficiency with addition and subtraction within 20 (1.OA.C.6) and their growing proficiency with multidigit addition and subtraction (1.NBT) and can involve easier and more accurate forward methods.[15]
- Word problems can also be linked to students' growing understanding of properties of addition and the relationship between addition and subtraction. For example, put-together/take-apart problems with unknown addends can show subtraction as finding an unknown addend (see the "Problem Types" section).[16]
- Units are a connection between place value (1.NBT) and measurement (1.MD). Working with place value depends on having a sense of the sizes of the base-ten units and being able to see a larger unit as composed of smaller units within the system. As measurement develops through the grades, measurement also depends on having a sense of the sizes of units and being able to see a larger unit as composed of smaller units within the system. In later grades, unit thinking will become important throughout arithmetic, including in the development of multidigit multiplication and division algorithms and the development of fraction concepts and operations.[17]
- Measurement standards 1.MD.A.1 and 1.MD.A.2 together support and provide a context for the 1.OA.A.1 goal of solving problems that involve comparing. To meet 1.MD.A.1, students compare the lengths of two objects by means of a third object, e.g., a length of string, that allows a "copy" of the length of an immovable object to be moved to another location to compare with the length of a movable object. When students cannot find the ***exact*** difference because of the magnitude of the numbers that arise from measurement—as may occur in comparing two students' heights—they may still compare the measurements to know which is greater (1.NBT.B.3). (Grade 2 standard 2.MD.B.6 formalizes this idea on a number-line diagram.)

Measuring a hallway using students as length units (1.MD.A.2). (The students are posited as having equal heights.)

- While students are dealing with the limited precision of only whole hours and half-hours, they must distinguish the position of the hour hand and connect it to the geometry standard 1.G.A.3, partitioning circles into halves and quarters.
- Composing shapes to create a new shape (1.G.A.2) is the spatial analogue of composing numbers to create new numbers. This concept is also connected to length measurement (1.MD.A.2) since students must visualize an object that is to be measured as being built up out of equal-sized units (see also 1.G.A.3). Though assembling two congruent right triangles into a rectangle does not use the same facts or reasoning that assembling two fives into a ten uses, the

[15] See page 13 of the *K–5 Progressions Document for Counting and Cardinality and Operations and Algebraic Thinking*, available at http://math.arizona.edu/~ime/progressions/ (2014).
[16] See page 13 of the *K–5 Progressions Document for Counting and Cardinality and Operations and Algebraic Thinking*, available at http://math.arizona.edu/~ime/progressions/ (2014).
[17] See *Units, a Unifying Idea in Measurement, Fractions, and Base Ten*, available at http://commoncoretools.me/2013/04/19/units-a-unifying-idea/ (2013).

idea of looking at how objects in some domain (numbers or shapes) can be combined to make other objects in that domain, and looking for new true statements one can make about these combinations, is a big idea that is common across mathematics.

Examples of Opportunities for In-Depth Focus

1.NBT.B.2 Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases: 10 can be thought of as a bundle of ten ones—called a "ten" (1.NBT.B.2.A); The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones (1.NBT.B.2.B); The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones) (1.NBT.B.2.C).

Grade 1 is students' first encounter with the three linked components of the place-value system: base ten units of tens and ones, composing and decomposing (bundling and unbundling) of units, and positional place-value notation. Understanding these and their connections is the foundation of the entire NBT domain.

1.NBT.C.4 Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.

Understanding place value is not a final goal on its own; the goal is to use place-value understanding and properties of operations to add and subtract (cluster 1.NBT.B). Students learn how standard place-value notation presents units of ten and units of one, and learn how to extend their single-digit methods for adding units of ones to add units of tens, including separately counting or adding the ones and counting or adding the tens in concrete models or drawings. When adding two arbitrary two-digit numbers (with a result within 100), being able to represent this addition with objects or drawings that show the base-ten structure supports students to understand when they need to compose a ten and how to record that new ten in their written method. Having a strong mental image and ability with single-digit versions of these additions and understanding how place-value notation can record these additions is the foundation for the extension of place-value reasoning and adding of separate units to grade 2 adding of tens that require composing a hundred (e.g., in 65 + 78) and adding units of hundreds (e.g., in 265 + 478). Note that students only subtract multiples of ten (e.g., 80 – 30) in grade 1 (1.NBT.C.6) because solving problems that do not require decomposing a ten (e.g., 78 – 32) for a long time before solving problems that do require decomposing a ten can lead to the common top-from-bottom error (e.g., 73 – 49 = 36).

1.OA.A.1 Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem

There are many distinct elementary addition and subtraction situations; students in grade 1 should work extensively with all of them.[18]

Examples of Opportunities for Connecting Mathematical Content and Mathematical Practices

Mathematical practices should be evident throughout mathematics instruction and should be centrally connected to the most important work of the grade. These mathematical practices can be summarized in the following sentence: Teachers help students to do mathematical sense-making about mathematical structure using mathematical drawings (or objects) to support mathematical explaining (MP 1 & 6; 7 & 8; 4 & 5; 2 & 3). Some brief examples of how the content of this grade might be connected to the Standards for Mathematical Practice follow.

- All work with properties (1.OA.B.3) and with understanding and using place value (e.g., 1.NBT.B.2, 1.NBT.C.4) should be seen as an investigation and use of the structure of the number system and of arithmetic (MP.7). Students' explanations of the properties and reasoning that they used in these contexts are early beginnings of the construction of (brief) logical arguments (MP.3). Examples of brief but excellent arguments at this grade level could include:
    - I know that 7 – 3 equals 4 because 4 + 3 equals 7. (This shows 1.OA.B.4 being met.)
    - I knew that 8 + 8 = 20 was wrong because 10 + 10 equals 20 and 8 is less than 10.
    - I know that 8 + 7 equals 15 because I know that 8 + 8 equals 16.
- Use of MP.8 ("Look for and express regularity in repeated reasoning") is important in the work on adding two-digit numbers (1.NBT.C.4), as described in the preceding in-depth-focus section. Students will repeatedly think about the units of ten and the units of one in their concrete models or drawings and in their recorded written methods. This work also uses MP.4 (modeling with mathematics), providing an example of a common situation in which multiple practices are involved in a given activity.

Manipulatives such as physical models of hundreds, tens, and ones, and visual models such as math drawings, are important parts of the K–2 classroom. These manipulatives and visual models should always be connected to written symbols and methods.

- Students in grade 1 work with some sophisticated addition and subtraction situations (1.OA.A.1), such as "Lucy has 8 fewer apples than Julie. Julie has 12 apples. How many apples does Lucy have?" Making a math drawing or using objects to model the situation is very helpful for students. The equations 12 – 8 = ?, 8 + ? = 12, and ? + 8 = 12 are all mathematical models of this situation (MP.4).

[18] Some situation subtypes need not be mastered until grade 2. See Table 2 on page 9 of the *K–5 Progressions Document for Counting and Cardinality and Operations and Algebraic Thinking*, available at http://math.arizona.edu/~ime/progressions/ (2014).

A note on manipulatives in grades K–2: Manipulatives such as physical models of hundreds, tens, and ones, and visual models such as math drawings, are important parts of the K–2 classroom. These manipulatives and visual models should always be connected to written symbols and methods.[19]

Content Emphases by Cluster

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The following table identifies the Major Clusters, Supporting Clusters, and Additional Clusters for each grade; below the table, suggestions are given for ways to connect the Supporting Clusters to the Major Clusters of the grade. Thus, rather than suggesting, even inadvertently, that some material should not be taught, this document provides direct advice for teaching grade-level material in ways that foster greater focus and coherence.

Key: ■ Major Clusters; ◘ Supporting Clusters; ⬡ Additional Clusters

Operations and Algebraic Thinking

- ■ Represent and solve problems involving addition and subtraction.
- ■ Understand and apply properties of operations and the relationship between addition and subtraction.
- ■ Add and subtract within 20.
- ■ Work with addition and subtraction equations.

Number and Operations in Base Ten

- ■ Extend the counting sequence.

[19] See page 19 of the *K–8 Publishers' Criteria for the Common Core State Standards for Mathematics*, available at http://www.corestandards.org/assets/Math_Publishers_Criteria_K-8_Summer%202012_FINAL.pdf. (2014)

- Understand place value.
- Use place value understanding and properties of operations to add and subtract.

Measurement and Data

- Measure lengths indirectly and by iterating length units.
- Tell and write time.
- Represent and interpret data.

Geometry

- Reason with shapes and their attributes.

Examples of Linking Supporting Clusters to the Major Work of the Grade

- When students work with organizing, representing, and interpreting data, the process includes practicing using numbers and adding and subtracting to answer questions about the data (see the part of 1.MD.C.4 after the semicolon (". . . ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another"), and see the K–5 progressions document for Measurement and Data (data part), especially Table 1 on page 4 and the discussion of categorical data on pages 5 and 6).[20]
- Telling and writing time on digital clocks (1.MD.B.3) is a context in which one can practice reading numbers (1.NBT.A.1), a kind of "application," but no more. Relating those times to ***meanings***—such as events during a day—is not part of 1.MD.B.3, but making sense of what one is doing (MP.1) and contextualizing (MP.2) are essential elements of good mathematical practice and should be part of the instructional foreground at all times.

[20] The *K–5 Progressions Document for Measurement and Data* (data part) is available at http://commoncoretools.files.wordpress.com/2011/06/ccss_progression_md_k5_2011_06_20.pdf (2011).

Examples of Key Advances from Grade 1 to Grade 2

- Students read and write numbers through 1,000, extending their use of place value to include units of hundreds.
- Students use their understanding of place value to add and subtract within 1,000 (e.g., 237 + 616 or 822 – 237). They can explain what they are doing as they add and subtract, and record their written method, using visual models to support calculating and explaining. They become fluent in addition and subtraction within 100.
- For word problems, students extend their ability by solving two-step problems using addition, subtraction, or both operations. They also master more advanced one-step addition and subtraction problems in this grade (such as take from with start unknown).[21]
- Students use standard units of measure and appropriate measurement tools. They understand basic properties of linear measurement (e.g., length, distance), such as the fact that the smaller the unit, the more iterations will be needed to cover a given length.

Fluency Expectations or Examples of Culminating Standards

2.OA.B.2 Fluently add and subtract within 20 using mental strategies. By end of grade 2, know from memory all sums of two one-digit numbers.

2.NBT.B.5 Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.

Critical area 2 within the grade 2 CCSS for Mathematics introduction says "They solve problems within 1000 by applying their understanding of models for addition and subtraction, and they develop, discuss, and use efficient, accurate, and generalizable methods to compute sums and differences of whole numbers in base-ten notation, using their understanding of place value and the properties of operations." As a result, students also use efficient, accurate, and generalizable methods for fluency within 100.

2.NBT.A.2 Count within 1000; skip-count by 5s, 10s, and 100s.

2.NBT.A.3 Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.

Fluency with these relationships is important for adding and subtracting within 1000.

[21] See Table 2 on page 9 of the *K–5 Progressions Document for Counting and Cardinality and Operations and Algebraic Thinking*, available at http://math.arizona.edu/~ime/progressions/ (2014). This table is based on Table 1 on page 88 of the CCSS for Mathematics.

2.NBT.B.8 Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900.

2.MD.A.4 Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.

Students require sufficient practice to measure accurately and reasonably quickly.

Examples of Major Within-Grade Dependencies

- "Understand place value" (cluster 2.NBT.A) is the foundation for "Use place value understanding and the properties of operations to add and subtract" (cluster 2.NBT.B). (Mastery of the two clusters can grow over time, in tandem with each other.) Adding and subtracting within 1,000 (2.NBT.B.7) involves adding or subtracting hundreds with hundreds, tens with tens, and ones with ones, sometimes composing or decomposing tens or hundreds. These ideas and methods rest on an understanding of the place-value units (2.NBT.A.1, building on 1.NBT.A.2) and understanding these units deepens students' understanding of place value.
- Knowing single-digit sums from memory (2.OA.B.2) is the basis for adding and subtracting multidigit numbers fluently and efficiently in general (cluster 2.NBT.B).

Examples of Opportunities for Connections among Standards, Clusters, or Domains

- Problems involving dollars, dimes, and pennies (2.MD.C.8) should be connected with the place-value learning of hundreds, tens, and ones (2.NBT.A.1), though the notation is different. A dollar is 100 cents, or a "bundle" of 10 dimes, each of which is a "bundle" of 10 pennies. Work with dollars, dimes, and pennies (without the notation) can support methods of whole-number addition (e.g., dimes are added to dimes). Addition that is appropriate with whole numbers can be explored in the new notation of money contexts (though fluency with that notation is ***not*** a standard at this grade).
- Students' work with addition and subtraction word problems (2.OA.A.1) can be coordinated with their growing skill in multidigit addition and subtraction (2.OA.B.2; cluster 2.NBT.B).
- Work with nickels (2.MD.C.8) and with telling time to the nearest five minutes on analog clocks (2.MD.C.7) should be taken together with counting by 5s (2.NBT.A.2) as contexts for gaining familiarity with repeating groups of 5 (2.OA.C.4). Recognizing time by seeing the minute hand at 3 and ***knowing*** that that signifies 15 minutes; recognizing three nickels as 15 cents; and seeing the 15-ness of a 3-by-5 rectangular array held in any position (including with neither base horizontal) will prepare for understanding, in grade 3, what the new operation of multiplication means.
- A number line (2.MD.B.6) connects numbers, lengths, and units. Number lines are first used in grade 2. A number line shows units of length; the numbers at the end points of the lengths tell how many lengths so far. Bar-graph scales (2.MD.D.10) and rulers (2.MD.A.1, 2, 3, 4) are number lines. Length units can be added and subtracted using rulers or number-line diagrams (2.MD.B.5, 6); adding lengths is an extension of adding and subtracting numbers of things, which has been a

focus in kindergarten and grade 1 and will be a focus in grade 2 OA and NBT standards. The purpose of number lines is to represent numbers, sums, and differences as lengths, rather than using lengths to solve all addition and subtraction problems.

Examples of Opportunities for In-Depth Focus

2.OA.A.1 Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem

Using situations (from word problems, from classroom events or student experiences, and from discovered mathematical patterns) as a source of problems can help students make sense of and contextualize the operations they are learning. Students continue to relate the different basic situation types in addition and subtraction.[22] Using equations and drawn models to represent situations can facilitate understanding, explaining, and such relating.

2.NBT.B.7 Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.

It takes substantial time for students to extend addition and subtraction to 1,000. Students must connect steps in the written method to what they know about place value and properties of operations, using visual models to support sense-making and explaining. Students need to be able to use general methods that will extend to larger numbers in grade 4, so that grade 3 students can concentrate on multiplication and division and on becoming fluent in their grade 2 addition and subtraction written methods within 1000.

Examples of Opportunities for Connecting Mathematical Content and Mathematical Practice

Mathematical practices should be evident throughout mathematics instruction and should be centrally connected to the most important work of the grade. These mathematical practices can be summarized in the following sentence: Teachers help students to do mathematical sense-making about mathematical structure using mathematical drawings (or objects) to support mathematical explaining (MP 1 & 6; 7 & 8; 4 & 5; 2 & 3). Some brief examples of how the content of this grade might be connected to the Standards for Mathematical Practice follow.

[22] See Table 2 on page 9 of the *K–5 Progressions Document for Counting and Cardinality and Operations and Algebraic Thinking*, available at http://math.arizona.edu/~ime/progressions/ (2014); as well as *Representing and Solving Addition and Subtraction Problems Mini-Assessment*, available at http://www.achievethecore.org/page/258/representing-and-solving-addition-and-subtraction-problems (2014).

- Students use MP.7 ("Look for and make use of structure") as they compose ones and tens to make tens and hundreds, and decompose hundreds and tens when they need more tens and ones. They use MP.8 ("Look for and express regularity in repeated reasoning") both as they compose and decompose such units and as they initially extend their reasoning and written methods from adding and subtracting within 100 to adding and subtracting within 1000.
- Grade 2 students use objects that remain appropriate tools (MP.5) for a lifetime: rulers, clocks, coins, and the number line (essentially an abstract ruler or measurement scale). MP.5 is about not just the ability to use tools, but the ability to choose the appropriate tool for a task. At this grade level, because students are just beginning to use a variety of tools, their utility may seem both obvious and fixed to a task: for example, a ruler measures length. For some students, for example, coming to understand the significance of counting by 5—the usefulness of that counting sequence and the situations in which it appears—may involve a choice of which of several images (nickels, hands, telling time) is most clarifying and salient to them. Generating the abstraction—in this example, the sequence 0, 5, 10, 15, etc.—may also be aided by experiences in the various domains, including the recognition that the same sequence of numbers is common to all of them. That sequence of number names expresses the regularity (MP.8) of a calculation (counting five more) that recurs in many contexts.

Manipulatives such as physical models of hundreds, tens, and ones, and visual models such as math drawings, are important parts of the K–2 classroom. These manipulatives and visual models should always be connected to written symbols and methods.

A note on manipulatives in grades K–2: Manipulatives such as physical models of hundreds, tens, and ones, and visual models such as math drawings, are important parts of the K–2 classroom. These manipulatives and visual models should always be connected to written symbols and methods.[23]

Content Emphases by Cluster

Not all of the content in a given grade is emphasized equally in the standards. Some clusters require greater emphasis than others, based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness. In addition, an intense focus on the most critical material at each grade allows depth in learning, which is carried out through the Standards for Mathematical Practice.

To say that some topics have greater emphasis is not to say that any content in the standards can safely be neglected in instruction. Neglecting material will leave gaps in student skill and understanding and may leave students unprepared for the challenges of a later grade. All standards figure in a

[23] See page 19 of *K–8 Publishers' Criteria for the Common Core State Standards for Mathematics*, available at http://www.corestandards.org/assets/Math_Publishers_Criteria_K-8_Summer%202012_FINAL.pdf. (2014)

mathematical education and will therefore be eligible for inclusion on PARCC assessments. However, the assessments will strongly focus where the standards strongly focus.

The following table identifies the Major Clusters, Supporting Clusters, and Additional Clusters for each grade; below the table, suggestions are given for ways to connect the Supporting Clusters to the Major Clusters of the grade. Thus, rather than suggesting, even inadvertently, that some material should not be taught, this document provides direct advice for teaching grade-level material in ways that foster greater focus and coherence.

Key: ■ Major Clusters; ▢ Supporting Clusters; ⬡ Additional Clusters

Operations and Algebraic Thinking

■ Represent and solve problems involving addition and subtraction.

■ Add and subtract within 20.

▢ Work with equal groups of objects to gain foundations for multiplication.

Number and Operations in Base Ten

■ Understand place value.

■ Use place value understanding and properties of operations to add and subtract.

Measurement and Data

■ Measure and estimate lengths in standard units.

■ Relate addition and subtraction to length.

▢ Work with time and money.

▢ Represent and interpret data.

Geometry

⬡ Reason with shapes and their attributes.

Examples of Linking Supporting Clusters to the Major Work of the Grade

- When students work with time and money (cluster 2.MD.C), their work with dollars, dimes, and pennies should support their understanding and skill in place value (2.NBT). Their work with nickels, with telling time to the nearest five minutes on analog clocks, with counting by 5s (2.NBT.A.2), and with arrays of five rows and/or five columns (cluster 2.OA.C) should be related.
- In cluster 2.MD.D ("Represent and interpret data"), standard 2.MD.D.10 represents an opportunity to link to the major work of grade 2. Picture graphs and bar graphs can add variety as contexts for posing and solving addition and subtraction problems. The language in 2.MD.D.10 mentions word problems (2.OA) explicitly. See the K–5 progressions document for Measurement and Data (data part).[24]

[24] For more on the connections between data work and arithmetic in the early grades see *K–5 Progressions Document for Measurement and Data* (data part) is available at http://commoncoretools.files.wordpress.com/2011/06/ccss_progression_md_k5_2011_06_20.pdf (2011).