Examples of Key Advances from Grade 2 to Grade 3

- Students in grade 3 begin to enlarge their concept of number by developing an understanding of fractions as numbers. This work will continue in grades 3-6, preparing the way for work with the complete rational number system in grades 6 and 7.

- Students in grades K-2 worked on number; place value; and addition and subtraction concepts, skills and problem solving. Beginning in grade 3, students will learn concepts, skills, and problem solving for multiplication and division. This work will continue in grades 3, 4 and 5, preparing the way for work with ratios and proportions in grades 6 and 7.

Fluency Expectations or Examples of Culminating Standards

3.OA.C.7 Students fluently multiply and divide within 100. By the end of grade 3, they know all products of two one-digit numbers from memory.

3.NBT.A.2 Students fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction. (Although 3.OA.C.7 and 3.NBT.A.2 are both fluency standards, these two standards do not represent equal investments of time in grade 3. Note that students in grade 2 were already adding and subtracting within 1000, just not fluently. That makes 3.NBT.A.2 a relatively small and incremental expectation. By contrast, multiplication and division are new in grade 3, and meeting the multiplication and division fluency standard 3.OA.C.7 with understanding is a major portion of students' work in grade 3.)

Examples of Major Within-Grade Dependencies

- Students must begin work with multiplication and division (3.OA) at or near the very start of the year to allow time for understanding and fluency to develop. Note that area models for products are an important part of this process (3.MD.C.7). Hence, work on concepts of area (3.MD.C.5–6) should likely begin at or near the start of the year as well.

Examples of Opportunities for Connections among Standards, Clusters or Domains

- Students' work with partitioning shapes (3.G.A.2) relates to visual fraction models (3.NF).

- Scaled picture graphs and scaled bar graphs (3.MD.B.3) can be a visually appealing context for solving multiplication and division problems.

Examples of Opportunities for In-Depth Focus

3.OA.A.3 Word problems involving equal groups, arrays, and measurement quantities can be used to build students' understanding of and skill with multiplication and division, as well as to allow students to demonstrate their understanding of and skill with these operations.

3.OA.C.7 Finding single-digit products and related quotients is a required fluency for grade 3. Reaching fluency will take much of the year for many students. These skills and the understandings that support them are crucial; students will rely on them for years to come as they learn to multiply and divide with multidigit whole numbers and to add, subtract, multiply, and divide with fractions. After multiplication and division situations have been established, reasoning about patterns in products (e.g., products involving factors of 5 or 9) can help students remember particular products and quotients. Practice — and if necessary, extra support — should continue all year for those who need it to attain fluency.

3.NF.A.2 Developing an understanding of fractions as numbers is essential for future work with the number system. It is critical that students at this grade are able to place fractions on a number line diagram and understand them as a related component of their ever-expanding number system.

3.MD.A.2 Continuous measurement quantities such as liquid volume, mass, and so on are an important context for fraction arithmetic (cf. 4.NF.B.4c, 5.NF.B.7c, 5.NF.B.3). In grade 3, students begin to get a feel for continuous measurement quantities and solve whole-number problems involving such quantities.

3.MD.C.7 Area is a major concept within measurement, and area models must function as a support for multiplicative reasoning in grade 3 and beyond.

Examples of Opportunities for Connecting Mathematical Content and Mathematical Practices

Mathematical practices should be evident *throughout* mathematics instruction and connected to all of the content areas highlighted above, as well as all other content areas addressed at this grade level. Mathematical tasks (short, long, scaffolded, and unscaffolded) are an important opportunity to connect content and practices. Some brief examples of how the content of this grade might be connected to the practices follow.

- Students learn and use strategies for finding products and quotients that are based on the properties of operations; for example, to find 4×7, they may recognize that $7 = 5 + 2$ and compute $4 \times 5 + 4 \times 2$. This is an example of seeing and making use of structure (MP.7). Such reasoning processes amount to brief arguments that students may construct and critique (MP.3).

- Students will analyze a number of situation types for multiplication and division, including arrays and measurement contexts. Extending their understanding of multiplication and division to these situations requires that they make sense of problems and persevere in solving them (MP.1), look for and make use of structure (MP.7) as they model these situations with mathematical forms (MP.4), and attend to precision (MP.6) as they distinguish different kinds of situations over time (MP.8).

Content Emphases by Cluster[8]

Not all of the content in a given grade is emphasized equally in the standards. Some clusters require greater emphasis than the others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness. In addition, an intense focus on the most critical material at each grade allows depth in learning, which is carried out through the Standards for Mathematical Practice.

To say that some things have greater emphasis is not to say that anything in the standards can safely be neglected in instruction. Neglecting material will leave gaps in student skill and understanding and may leave students unprepared for the challenges of a later grade. All standards figure in a mathematical education and will therefore be eligible for inclusion on the PARCC assessment. However, the assessments will strongly focus where the standards strongly focus.

In addition to identifying the Major, Additional, and Supporting Clusters for each grade, suggestions are given following the table on the next page for ways to connect the Supporting to the Major Clusters of the grade. Thus, rather than suggesting even inadvertently that some material not be taught, there is direct advice for teaching it, in ways that foster greater focus and coherence.

Key: ■ Major Clusters; ▣ Supporting Clusters; ○ Additional Clusters

Operations and Algebraic Thinking	
■	A. Represent and solve problems involving multiplication and division.
■	B. Understand properties of multiplication and the relationship between multiplication and division.
■	C. Multiply and divide within 100.
■	D. Solve problems involving the four operations, and identify and explain patterns in arithmetic.
Number and Operations in Base Ten	
○	A. Use place value understanding and properties of operations to perform multi-digit arithmetic.
Number and Operations — Fractions	
■	A. Develop understanding of fractions as numbers.
Measurement and Data	
■	A. Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.
▣	B. Represent and interpret data.
■	C. Geometric measurement: understand concepts of area and relate area to multiplication and addition.
○	D. Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.
Geometry	
▣	A. Reason with shapes and their attributes.

[8] Refer to pages 13–14 for further explanation of the cluster-level emphases. Refer also to the Common Core State Standards for Mathematics for the standards that fall within each cluster.

Examples of Linking Supporting Clusters to the Major Work of the Grade

- Represent and interpret data: Students multiply and divide to solve problems using information presented in scaled bar graphs (3.MD.B.3). Pictographs and scaled bar graphs are a visually appealing context for one- and two-step word problems.

- Reason with shapes and their attributes: Work toward meeting 3.G.A.2 should be positioned in support of area measurement and understanding of fractions.

Examples of Key Advances from Grade 3 to Grade 4

- In grade 3, students studied multiplication in terms of equal groups, arrays and area. In grade 4, students extend their concept of multiplication to make multiplicative comparisons (4.OA.A.1).[9]
- Students in grade 4 apply and extend their understanding of the meanings and properties of addition and subtraction of whole numbers to extend addition and subtraction to fractions (4.NF.B.3).[10]
- Fraction equivalence is an important theme within the standards that begins in grade 3. In grade 4, students extend their understanding of fraction equivalence to the general case, $a/b = (n \times a)/(n \times b)$ (3.NF.A.3 $\rightarrow$ 4.NF.A.1).[11] They apply this understanding to compare fractions in the general case (3.NF.A.3d $\rightarrow$ 4.NF.A.2).
- Students in grade 4 apply and extend their understanding of the meanings and properties of multiplication of whole numbers to multiply a fraction by a whole number (4.NF.B.4).
- Students in grade 4 begin using the four operations to solve word problems involving measurement quantities such as liquid volume, mass and time (4.MD.A.2).
- Students combine their understanding of the meanings and properties of multiplication and division with their understanding of base-ten units to begin to multiply and divide multidigit numbers (4.NBT.B.5–6; this builds on work done in grade 3, cf. 3.NBT.A.3).
- Students generalize their previous understanding of place value for multidigit whole numbers (4.NBT.A.1–3). This supports their work in multidigit multiplication and division, carrying forward into grade 5, when students will extend place value to decimals.

Fluency Expectations or Examples of Culminating Standards

4.NBT.B.4 Students fluently add and subtract multidigit whole numbers using the standard algorithm.

Examples of Major Within-Grade Dependencies

- Students' work with decimals (4.NF.C.5–7) depends to some extent on concepts of fraction equivalence and elements of fraction arithmetic. Students express fractions with a denominator of 10 as an equivalent fraction with a denominator of 100; comparisons of decimals require that students use similar reasoning to comparisons with fractions.

[9] In an additive comparison problem (grades 1–2), the underlying question is *what amount would be added to one quantity to result in the other?* In a multiplicative comparison problem, the underlying question is *what factor would multiply one quantity to result in the other?*

[10] This work is limited to equal denominators in grade 4 to give students more time to build their understanding of fraction equivalence, before adding and subtracting unlike denominators in grade 5.

[11] Students who can generate equivalent fractions can also develop strategies for adding fractions with different denominators, but this is not a requirement in grade 4.

- Standard 4.MD.A.2 refers to using the four operations to solve word problems involving measurement quantities such as liquid volume, mass, time, and so on. Some parts of this standard could be met earlier in the year (such as using whole-number multiplication to express measurements given in a larger unit in terms of a smaller unit — see also 4.MD.A.1), while others might be met only by the end of the year (such as word problems involving addition and subtraction of fractions or multiplication of a fraction by a whole number — see also 4.NF.B.3d and 4.NF.B.4c).

- Standard 4.MD.C.7 refers to word problems involving unknown angle measures. Before this standard can be met, students must understand concepts of angle measure (4.MD.C.5) and, presumably, gain some experience measuring angles (4.MD.C.6). Before that can happen, students must have some familiarity with the geometric terms that are used to define angles as geometric shapes (4.G.A.1).

Examples of Opportunities for Connections among Standards, Clusters or Domains

- The work that students do with units of measure (4.MD.A.1–2) and with multiplication of a fraction by a whole number (4.NF.B.4) can be connected to the idea of "times as much" in multiplication (4.OA.A.1).

- Addition of fractions (4.NF.B.3) and concepts of angle measure (4.MD.C.5a and 4.MD.C.7) are connected in that a one-degree measure is a fraction of an entire rotation and that adding angle measures together is adding fractions with a denominator of 360.

Examples of Opportunities for In-Depth Focus

4.NBT.B.5 When students work toward meeting this standard, they combine prior understanding of multiplication with deepening understanding of the base-ten system of units to express the product of two multidigit numbers as another multidigit number. This work will continue in grade 5 and culminate in fluency with the standard algorithms in grade 6.

4.NBT.B.6 When students work toward meeting this standard, they combine prior understanding of multiplication and division with deepening understanding of the base-ten system of units to find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors. This work will develop further in grade 5 and culminate in fluency with the standard algorithms in grade 6.

4.NF.A.1 Extending fraction equivalence to the general case is necessary to extend arithmetic from whole numbers to fractions and decimals.

4.NF.B.3 This standard represents an important step in the multigrade progression for addition and subtraction of fractions. Students extend their prior understanding of addition and subtraction to add and subtract fractions with like denominators by thinking of adding or subtracting so many unit fractions.

4.NF.B.4 This standard represents an important step in the multigrade progression for multiplication and division of fractions. Students extend their developing understanding of multiplication to multiply a fraction by a whole number.

Examples of Opportunities for Connecting Mathematical Content and Mathematical Practices

Mathematical practices should be evident *throughout* mathematics instruction and connected to all of the content areas highlighted above, as well as all other content areas addressed at this grade level. Mathematical tasks (short, long, scaffolded, and unscaffolded) are an important opportunity to connect content and practices. Some brief examples of how the content of this grade might be connected to the practices follow.

- When students decompose numbers into sums of multiples of base-ten units to multiply them (4.NBT.B.5), they are seeing and making use of structure (MP.7). As they illustrate and explain the calculation by using physical or drawn models, they are modeling (MP.4), using appropriate drawn tools strategically (MP.5) and attending to precision (MP.6) as they use base-ten units in the appropriate places.

- To compute and interpret remainders in word problems (4.OA.A.3), students must reason abstractly and quantitatively (MP.2), make sense of problems (MP.1), and look for and express regularity in repeated reasoning (MP.8) as they search for the structure (MP.7) in problems with similar interpretations of remainders.

Content Emphases by Cluster[12]

Not all of the content in a given grade is emphasized equally in the standards. Some clusters require greater emphasis than the others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness. In addition, an intense focus on the most critical material at each grade allows depth in learning, which is carried out through the Standards for Mathematical Practice.

To say that some things have greater emphasis is not to say that anything in the standards can safely be neglected in instruction. Neglecting material will leave gaps in student skill and understanding and may leave students unprepared for the challenges of a later grade. All standards figure in a mathematical education and will therefore be eligible for inclusion on the PARCC assessment. However, the assessments will strongly focus where the standards strongly focus.

In addition to identifying the Major, Additional, and Supporting Clusters for each grade, suggestions are given following the table on the next page for ways to connect the Supporting to the Major Clusters of the grade. Thus, rather than suggesting even inadvertently that some material not be taught, there is direct advice for teaching it, in ways that foster greater focus and coherence.

[12] Refer to pages 13–14 for further explanation of the cluster-level emphases. Refer also to the Common Core State Standards for Mathematics for the standards that fall within each cluster.

Key: ■ Major Clusters; ◘ Supporting Clusters; ○ Additional Clusters

Operations and Algebraic Thinking

- ■ A. Use the four operations with whole numbers to solve problems.
- ◘ B. Gain familiarity with factors and multiples.
- ○ C. Generate and analyze patterns.

Number and Operations in Base Ten

- ■ A. Generalize place value understanding for multi-digit whole numbers.
- ■ B. Use place value understanding and properties of operations to perform multi-digit arithmetic.

Number and Operations — Fractions

- ■ A. Extend understanding of fraction equivalence and ordering.
- ■ B. Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.
- ■ C. Understand decimal notation for fractions, and compare decimal fractions.

Measurement and Data

- ◘ A. Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.
- ◘ B. Represent and interpret data.
- ○ C. Geometric measurement: understand concepts of angle and measure angles.

Geometry

- ○ A. Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

Examples of Linking Supporting Clusters to the Major Work of the Grade

- Gain familiarity with factors and multiples: Work in this cluster supports students' work with multidigit arithmetic as well as their work with fraction equivalence.

- Represent and interpret data: The standard in this cluster requires students to use a line plot to display measurements in fractions of a unit and to solve problems involving addition and subtraction of fractions, connecting it directly to the Number and Operations — Fractions clusters.

Examples of Key Advances from Grade 4 to Grade 5

- In grade 5, students will integrate decimal fractions more fully into the place value system (5.NBT.A.1–4). By thinking about decimals as sums of multiples of base-ten units, students begin to extend algorithms for multidigit operations to decimals (5.NBT.B.7).
- Students use their understanding of fraction equivalence and their skill in generating equivalent fractions as a strategy to add and subtract fractions, including fractions with unlike denominators.
- Students apply and extend their previous understanding of multiplication to multiply a fraction or whole number by a fraction (5.NF.B.4). They also learn the relationship between fractions and division, allowing them to divide any whole number by any nonzero whole number and express the answer in the form of a fraction or mixed number (5.NF.B.3). And they apply and extend their previous understanding of multiplication and division to divide a unit fraction by a whole number or a whole number by a unit fraction.[13]
- Students extend their grade 4 work in finding whole-number quotients and remainders to the case of two-digit divisors (5.NBT.B.6).
- Students continue their work in geometric measurement by working with volume as an attribute of solid figures and as a measurement quantity (5.MD.C.3–5).
- Students build on their previous work with number lines to use two perpendicular number lines to define a coordinate system (5.G.A.1–2).

Fluency Expectations or Examples of Culminating Standards

5.NBT.B.5 Students fluently multiply multidigit whole numbers using the standard algorithm.

Examples of Major Within-Grade Dependencies

- Understanding that in a multidigit number, a digit in one place represents $1/10$ of what it represents in the place to its left (5.NBT.A.1) is an example of multiplying a quantity by a fraction (5.NF.B.4).

Examples of Opportunities for Connections among Standards, Clusters or Domains

- The work that students do in multiplying fractions extends their understanding of the operation of multiplication. For example, to multiply $a/b \times q$ (where q is a whole number or a fraction), students can interpret $a/b \times q$ as meaning a parts of a partition of q into b equal parts (5.NF.B.4a). This interpretation of the product leads to a product that is less than, equal to or greater than q depending on whether $a/b < 1$, $a/b = 1$ or $a/b > 1$, respectively (5.NF.B.5).

[13] Students able to multiply fractions in general can develop strategies to divide fractions in general by reasoning about the relationship between multiplication and division. But the division of a fraction by a fraction is not a requirement in this grade.

- Conversions within the metric system represent an important practical application of the place value system. Students' work with these units (5.MD.A.1) can be connected to their work with place value (5.NBT.A.1).

Examples of Opportunities for In-Depth Focus

5.NBT.A.1 The extension of the place value system from whole numbers to decimals is a major intellectual accomplishment involving understanding and skill with base-ten units and fractions.

5.NBT.B.6 The extension from one-digit divisors to two-digit divisors requires care. This is a major milestone along the way to reaching fluency with the standard algorithm in grade 6 (6.NS.B.2).

5.NF.A.2 When students meet this standard, they bring together the threads of fraction equivalence (grades 3–5) and addition and subtraction (grades K–4) to fully extend addition and subtraction to fractions.

5.NF.B.4 When students meet this standard, they fully extend multiplication to fractions, making division of fractions in grade 6 (6.NS.A.1) a near target.

5.MD.C.5 Students work with volume as an attribute of a solid figure and as a measurement quantity. Students also relate volume to multiplication and addition. This work begins a progression leading to valuable skills in geometric measurement in middle school.

Examples of Opportunities for Connecting Mathematical Content and Mathematical Practices

Mathematical practices should be evident *throughout* mathematics instruction and connected to all of the content areas highlighted above, as well as all other content areas addressed at this grade level. Mathematical tasks (short, long, scaffolded, and unscaffolded) are an important opportunity to connect content and practices. Some brief examples of how the content of this grade might be connected to the practices follow.

- When students break divisors and dividends into sums of multiples of base-ten units (5.NBT.B.6), they are seeing and making use of structure (MP.7) and attending to precision (MP.6). Initially for most students, multidigit division problems take time and effort, so they also require perseverance (MP.1) and looking for and expressing regularity in repeated reasoning (MP.8).

- When students explain patterns in the number of zeros of the product when multiplying a number by powers of 10 (5.NBT.A.2), they have an opportunity to look for and express regularity in repeated reasoning (MP.8). When they use these patterns in division, they are making sense of problems (MP.1) and reasoning abstractly and quantitatively (MP.2).

- When students show that the volume of a right rectangular prism is the same as would be found by multiplying the side lengths (5.MD.C.5), they also have an opportunity to look for and express regularity in repeated reasoning (MP.8). They attend to precision (MP.6) as they use correct length or volume units, and they use appropriate tools strategically (MP.5) as they understand or make drawings to show these units.

Content Emphases by Cluster[14]

Not all of the content in a given grade is emphasized equally in the standards. Some clusters require greater emphasis than the others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness. In addition, an intense focus on the most critical material at each grade allows depth in learning, which is carried out through the Standards for Mathematical Practice.

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In addition to identifying the Major, Additional, and Supporting Clusters for each grade, suggestions are given following the table on the next page for ways to connect the Supporting to the Major Clusters of the grade. Thus, rather than suggesting even inadvertently that some material not be taught, there is direct advice for teaching it, in ways that foster greater focus and coherence.

Key: ■ Major Clusters; ■ Supporting Clusters; ○ Additional Clusters

Operations and Algebraic Thinking

- ○ A. Write and interpret numerical expressions.
- ○ B. Analyze patterns and relationships.

Number and Operations in Base Ten

- ■ A. Understand the place value system.
- ■ B. Perform operations with multi-digit whole numbers and with decimals to hundredths.

Number and Operations — Fractions

- ■ A. Use equivalent fractions as a strategy to add and subtract fractions
- ■ B. Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

Measurement and Data

- ■ A. Convert like measurement units within a given measurement system.
- ■ B. Represent and interpret data.
- ■ C. Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.

Geometry

- ○ A. Graph points on the coordinate plane to solve real-world and mathematical problems.
- ○ B. Classify two-dimensional figures into categories based on their properties.

[14] Refer to pages 13–14 for further explanation of the cluster-level emphases. Refer also to the Common Core State Standards for Mathematics for the standards that fall within each cluster.

Examples of Linking Supporting Clusters to the Major Work of the Grade

- Convert like measurement units within a given measurement system: Work in these standards supports computation with decimals. For example, converting 5 cm to 0.05 m involves computation with decimals to hundredths.

- Represent and interpret data: The standard in this cluster provides an opportunity for solving real-world problems with operations on fractions, connecting directly to both Number and Operations — Fractions clusters.

Examples of Key Advances from Grade 5 to Grade 6

- Students' prior understanding of and skill with multiplication, division, and fractions contribute to their study of ratios, proportional relationships and unit rates (6.RP).
- Students begin using properties of operations systematically to work with variables, variable expressions, and equations (6.EE).
- Students extend their work with the system of rational numbers to include using positive and negative numbers to describe quantities (6.NS.C.5), extending the number line and coordinate plane to represent rational numbers and ordered pairs (6.NS.C.6), and understanding ordering and absolute value of rational numbers (6.NS.C.7).
- Having worked with measurement data in previous grades, students begin to develop notions of statistical variability, summarizing and describing distributions (6.SP).

Fluency Expectations or Examples of Culminating Standards

6.NS.B.2 Students fluently divide multidigit numbers using the standard algorithm. This is the culminating standard for several years' worth of work with division of whole numbers.

6.NS.B.3 Students fluently add, subtract, multiply, and divide multidigit decimals using the standard algorithm for each operation. This is the culminating standard for several years' worth of work relating to the domains of Number and Operations in Base Ten, Operations and Algebraic Thinking, and Number and Operations — Fractions.

6.NS.A.1 Students interpret and compute quotients of fractions and solve word problems involving division of fractions by fractions. This completes the extension of operations to fractions.

Examples of Major Within-Grade Dependencies

- Equations of the form $px = q$ (6.EE.B.7) are unknown-factor problems; the solution will sometimes be the quotient of a fraction by a fraction (6.NS.A.1).
- Solving problems by writing and solving equations (6.EE.B.7) involves not only an appreciation of how variables are used (6.EE.B.6) and what it means to solve an equation (6.EE.B.5) but also some ability to write, read, and evaluate expressions in which letters stand for numbers (6.EE.A.2).
- Students must be able to place rational numbers on a number line (6.NS.C.7) before they can place ordered pairs of rational numbers on a coordinate plane (6.NS.C.8). The former standard about ordering rational numbers is much more fundamental.

Examples of Opportunities for Connections among Standards, Clusters or Domains

- Students' work with ratios and proportional relationships (6.RP) can be combined with their work in representing quantitative relationships between dependent and independent variables (6.EE.C.9).

- Plotting rational numbers in the coordinate plane (6.NS.C.8) is part of analyzing proportional relationships (6.RP.A.3a, 7.RP.A.2) and will become important for studying linear equations (8.EE.C.8) and graphs of functions (8.F).[15]
- Students use their skill in recognizing common factors (6.NS.B.4) to rewrite expressions (6.EE.A.3).
- Writing, reading, evaluating, and transforming variable expressions (6.EE.A.1–4) and solving equations and inequalities (6.EE.B.7–8) can be combined with use of the volume formulas $V = lwh$ and $V = Bh$ (6.G.A.2).
- Working with data sets can connect to estimation and mental computation. For example, in a situation where there are 20 different numbers that are all between 8 and 10, one might quickly estimate the sum of the numbers as $9 \times 20 = 180$.

Examples of Opportunities for In-Depth Focus

6.RP.A.3 When students work toward meeting this standard, they use a range of reasoning and representations to analyze proportional relationships.

6.NS.A.1 This is a culminating standard for extending multiplication and division to fractions.

6.NS.C.8 When students work with rational numbers in the coordinate plane to solve problems, they combine and consolidate elements from the other standards in this cluster.

6.EE.A.3 By applying properties of operations to generate equivalent expressions, students use properties of operations that they are familiar with from previous grades' work with numbers — generalizing arithmetic in the process.

6.EE.B.7 When students write equations of the form $x + p = q$ and $px = q$ to solve real-world and mathematical problems, they draw on meanings of operations that they are familiar with from previous grades' work. They also begin to learn algebraic approaches to solving problems.[16]

Examples of Opportunities for Connecting Mathematical Content and Mathematical Practices

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[15] While not required by the standards, it might be considered valuable to expose students to time series data and to time graphs as an appealing way to work with rational numbers in the coordinate plane (6.NS.C.8). For example, students could create time graphs of temperature measured each hour over a 24-hour period in a place where, to ensure a strong connection to rational numbers, temperature values might cross from positive to negative during the night and back to positive the next day.

[16] For example, suppose Daniel went to visit his grandmother, who gave him $5.50. Then he bought a book costing $9.20 and had $2.30 left. To find how much money he had before visiting his grandmother, an algebraic approach leads to the equation $x + 5.50 – 9.20 = 2.30$. An arithmetic approach without using variables at all would be to begin with 2.30, then add 9.20, then subtract 5.50. This yields the desired answer, but students will eventually encounter problems in which arithmetic approaches are unrealistically difficult and algebraic approaches must be used.

content and practices. Some brief examples of how the content of this grade might be connected to the practices follow.

- Reading and transforming expressions involves seeing and making use of structure (MP.7). Relating expressions to situations requires making sense of problems (MP.1) and reasoning abstractly and quantitatively (MP.2).
- The sequence of steps in the solution of an equation is a logical argument that students can construct and critique (MP.3). Such arguments require looking for and making use of structure (MP.7) and, over time, expressing regularity in repeated reasoning (MP.8).
- Thinking about the point (1, *r*) in a graph of a proportional relationship with unit rate *r* involves reasoning abstractly and quantitatively (MP.2). The graph models with mathematics (MP.4) and uses appropriate tools strategically (MP.5).
- Area, surface area, and volume present modeling opportunities (MP.4) and require students to attend to precision with the types of units involved (MP.6).
- Students think with precision (MP.6) and reason quantitatively (MP.2) when they use variables to represent numbers and write expressions and equations to solve a problem (6.EE.B.6–7).
- Working with data gives students an opportunity to use appropriate tools strategically (MP.5). For example, spreadsheets can be powerful for working with a data set with dozens or hundreds of data points.

Content Emphases by Cluster[17]

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[17] Refer to pages 13–14 for further explanation of the cluster-level emphases. Refer also to the Common Core State Standards for Mathematics for the standards that fall within each cluster.

Key: ■ Major Clusters; ▣ Supporting Clusters; ○ Additional Clusters

Ratios and Proportional Relationships

■ A. Understand ratio concepts and use ratio reasoning to solve problems.

The Number System

■ A. Apply and extend previous understandings of multiplication and division to divide fractions by fractions.

○ B. Compute fluently with multi-digit numbers and find common factors and multiples.

■ C. Apply and extend previous understandings of numbers to the system of rational numbers.

Expressions and Equations

■ A. Apply and extend previous understandings of arithmetic to algebraic expressions.

■ B. Reason about and solve one-variable equations and inequalities.

■ C. Represent and analyze quantitative relationships between dependent and independent variables.

Geometry

▣ A. Solve real-world and mathematical problems involving area, surface area and volume.

Statistics and Probability

○ A. Develop understanding of statistical variability.

○ B. Summarize and describe distributions.

Examples of Linking Supporting Clusters to the Major Work of the Grade

- Solve real-world and mathematical problems involving area, surface area, and volume: In this cluster, students work on problems with areas of triangles and volumes of right rectangular prisms, which connects to work in the Expressions and Equations domain. In addition, another standard within this cluster asks students to draw polygons in the coordinate plane, which supports other work with the coordinate plane in The Number System domain.

Examples of Key Advances from Grade 6 to Grade 7

- In grade 6, students learned about negative numbers and the kinds of quantities they can be used to represent; they also learned about absolute value and ordering of rational numbers, including in real-world contexts. In grade 7, students will add, subtract, multiply, and divide within the system of rational numbers.

- Students grow in their ability to analyze proportional relationships. They decide whether two quantities are in a proportional relationship (7.RP.A.2a); they work with percents, including simple interest, percent increase and decrease, tax, markups and markdowns, gratuities and commission, and percent error (7.RP.A.3); they analyze proportional relationships and solve problems involving unit rates associated with ratios of fractions (e.g., if a person walks $1/2$ mile in each $1/4$ hour, the unit rate is the complex fraction ½ / ¼ miles per hour or 2 miles per hour) (7.RP.A.1); and they analyze proportional relationships in geometric figures (7.G.A.1).

- Students solve a variety of problems involving angle measure, area, surface area, and volume (7.G.B.4–6).

Fluency Expectations or Examples of Culminating Standards

7.EE.B.3 Students solve multistep problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. This work is the culmination of many progressions of learning in arithmetic, problem solving and mathematical practices.

7.EE.B.4 In solving word problems leading to one-variable equations of the form $px + q = r$ and $p(x + q) = r$, students solve the equations fluently. This will require fluency with rational number arithmetic (7.NS.A.1–3), as well as fluency to some extent with applying properties operations to rewrite linear expressions with rational coefficients (7.EE.A.1).

7.NS.A.1–2 Adding, subtracting, multiplying, and dividing rational numbers is the culmination of numerical work with the four basic operations. The number system will continue to develop in grade 8, expanding to become the real numbers by the introduction of irrational numbers, and will develop further in high school, expanding to become the complex numbers with the introduction of imaginary numbers. Because there are no specific standards for rational number arithmetic in later grades and because so much other work in grade 7 depends on rational number arithmetic (see below), fluency with rational number arithmetic should be the goal in grade 7.

Examples of Major Within-Grade Dependencies

- Meeting standard 7.EE.B.3 in its entirety will involve using rational number arithmetic (7.NS.A.1–3) and percents (7.RP.A.3). Work leading to meeting this standard could be organized as a recurring activity that tracks the students' ongoing acquisition of new skills in rational number arithmetic and percents.

- Because rational number arithmetic (7.NS.A.1–3) underlies the problem solving detailed in 7.EE.B.3 as well as the solution of linear expressions and equations (7.EE.A.1–2, 4), this work should likely begin at or near the start of the year.

- The work leading to meeting standards 7.EE.A.1–4 could be divided into two phases, one centered on addition and subtraction (e.g., solving $x + q = r$) in relation to rational number addition and subtraction (7.NS.A.1) and another centered on multiplication and division (e.g., solving $px + q = r$ and $p(x + q) = r$) in relation to rational number multiplication and division (7.NS.A.2).

Examples of Opportunities for Connections among Standards, Clusters or Domains

- Students use proportional reasoning when they analyze scale drawings (7.G.A.1).

- Students use proportional reasoning and percentages when they extrapolate from random samples and use probability (7.SP.C.6, 8).

Examples of Opportunities for In-Depth Focus

7.RP.A.2 Students in grade 7 grow in their ability to recognize, represent, and analyze proportional relationships in various ways, including by using tables, graphs, and equations.

7.NS.A.3 When students work toward meeting this standard (which is closely connected to 7.NS.A.1 and 7.NS.A.2), they consolidate their skill and understanding of addition, subtraction, multiplication and division of rational numbers.

7.EE.B.3 This is a major capstone standard for arithmetic and its applications.

7.EE.B.4 Work toward meeting this standard builds on the work that led to meeting 6.EE.B.7 and prepares students for the work that will lead to meeting 8.EE.C.7.

7.G.B.6 Work toward meeting this standard draws together grades 3–6 work with geometric measurement.

Examples of Opportunities for Connecting Mathematical Content and Mathematical Practices

Mathematical practices should be evident *throughout* mathematics instruction and connected to all of the content areas highlighted above, as well as all other content areas addressed at this grade level. Mathematical tasks (short, long, scaffolded, and unscaffolded) are an important opportunity to connect content and practices. Some brief examples of how the content of this grade might be connected to the practices follow.

- When students compare arithmetic and algebraic solutions to the same problem (7.EE.B.4a), they are identifying correspondences between different approaches (MP.1).
- Solving an equation such as $4 = 8(x - 1/2)$ requires students to see and make use of structure (MP.7), temporarily viewing $x - 1/2$ as a single entity.
- When students notice when given geometric conditions determine a unique triangle, more than one triangle or no triangle (7.G.A.2), they have an opportunity to construct viable arguments and critique the reasoning of others (MP.3). Such problems also present opportunities for using appropriate tools strategically (MP.5).
- Proportional relationships present opportunities for modeling (MP.4). For example, the number of people who live in an apartment building might be taken as proportional to the number of stories in the building for modeling purposes.

Content Emphases by Cluster[18]

Not all of the content in a given grade is emphasized equally in the standards. Some clusters require greater emphasis than the others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness. In addition, an intense focus on the most critical material at each grade allows depth in learning, which is carried out through the Standards for Mathematical Practice.

To say that some things have greater emphasis is not to say that anything in the standards can safely be neglected in instruction. Neglecting material will leave gaps in student skill and understanding and may leave students unprepared for the challenges of a later grade. All standards figure in a mathematical education and will therefore be eligible for inclusion on the PARCC assessment. However, the assessments will strongly focus where the standards strongly focus.

In addition to identifying the Major, Additional, and Supporting Clusters for each grade, suggestions are given following the table on the next page for ways to connect the Supporting to the Major Clusters of the grade. Thus, rather than suggesting even inadvertently that some material not be taught, there is direct advice for teaching it, in ways that foster greater focus and coherence.

Key: ■ Major Clusters; ■ Supporting Clusters; ○ Additional Clusters

Ratios and Proportional Relationships

■ A. Analyze proportional relationships and use them to solve real-world and mathematical problems.

The Number System

■ A. Apply and extend previous understandings of operations with fractions to add, subtract, multiply and divide rational numbers.

Expressions and Equations

■ A. Use properties of operations to generate equivalent expressions.

■ B. Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

[18] Refer to pages 13–14 for further explanation of the cluster-level emphases. Refer also to the Common Core State Standards for Mathematics for the standards that fall within each cluster.

Geometry

- A. Draw, construct and describe geometrical figures and describe the relationships between them.
- B. Solve real-life and mathematical problems involving angle measure, area, surface area and volume.

Statistics and Probability

- A. Use random sampling to draw inferences about a population.
- B. Draw informal comparative inferences about two populations.
- C. Investigate chance processes and develop, use, and evaluate probability models.

Examples of Linking Supporting Clusters to the Major Work of the Grade

- Use random sampling to draw inferences about a population: The standards in this cluster represent opportunities to apply percentages and proportional reasoning. To make inferences about a population, one needs to apply such reasoning to the sample and the entire population.

- Investigate chance processes and develop, use, and evaluate probability models: Probability models draw on proportional reasoning and should be connected to the major work in those standards.

Examples of Key Advances from Grade 7 to Grade 8

- Students build on previous work with proportional relationships, unit rates, and graphing to connect these ideas and understand that the points (x, y) on a nonvertical line are the solutions of the equation $y = mx + b$, where m is the slope of the line as well as the unit rate of a proportional relationship (in the case $b = 0$). Students also formalize their previous work with linear relationships by working with functions — rules that assign to each input exactly one output.

- By working with equations such as $x^2 = 2$ and in geometric contexts such as the Pythagorean theorem, students enlarge their concept of number beyond the system of rationals to include irrational numbers. They represent these numbers with radical expressions and approximate these numbers with rationals.

Fluency Expectations or Examples of Culminating Standards

8.EE.C.7 Students have been working informally with one-variable linear equations since as early as kindergarten. This important line of development culminates in grade 8 with the solution of general one-variable linear equations, including cases with infinitely many solutions or no solutions as well as cases requiring algebraic manipulation using properties of operations. Coefficients and constants in these equations may be any rational numbers.

8.G.C.9 When students learn to solve problems involving volumes of cones, cylinders, and spheres — together with their previous grade 7 work in angle measure, area, surface area and volume (7.G.B.4–6) — they will have acquired a well-developed set of geometric measurement skills. These skills, along with proportional reasoning (7.RP) and multistep numerical problem solving (7.EE.B.3), can be combined and used in flexible ways as part of modeling during high school — not to mention after high school for college and careers.[19]

Examples of Major Within-Grade Dependencies

- An important development takes place in grade 8 when students make connections between proportional relationships, lines, and linear equations (8.EE, **second cluster**). Making these connections depends on prior grades' work, including 7.RP.A.2 and 6.EE.C.9. There is also a major dependency within grade 8 itself: The angle-angle criterion for triangle similarity underlies the fact that a nonvertical line in the coordinate plane has equation $y = mx + b$.[20] Therefore, students must do work with congruence and similarity (8.G.A.1–5) before they are able to justify the connections among proportional relationships, lines, and linear equations. Hence the indicated geometry work should likely begin at or near the very start of the year.[21]

[19] See "Appendix A: Lasting Achievements in K–8."

[20] See page 12 of the *Progression for Expressions and Equations:* http://commoncoretools.files.wordpress.com/2011/04/ccss_progression_ee_2011_04_25.pdf.

[21] Note that the Geometry cluster "Understand congruence and similarity using physical models, transparencies or geometry software" supports high school work with congruent triangles and congruent figures.

- Much of the work of grade 8 involves lines, linear equations, and linear functions (8.EE.B.5–8; 8.F.A.3–4; 8.SP.A.2–3). Irrational numbers, radicals, the Pythagorean theorem, and volume (8.NS.A.1–2; 8.EE.A.2; 8.G.B.6–9) are nonlinear in nature. Curriculum developers might choose to address linear and nonlinear bodies of content somewhat separately. An exception, however, might be that when addressing functions, pervasively treating linear functions as separate from nonlinear functions might obscure the concept of function *per se*. There should also be sufficient treatment of nonlinear functions to avoid giving students the misleading impression that all functional relationships are linear (see also 7.RP.A.2a).

Examples of Opportunities for Connections among Standards, Clusters or Domains

- Students' work with proportional relationships, lines, linear equations, and linear functions can be enhanced by working with scatter plots and linear models of association in bivariate measurement data (8.SP.A.1–3).

Examples of Opportunities for In-Depth Focus

8.EE.B.5 When students work toward meeting this standard, they build on grades 6–7 work with proportions and position themselves for grade 8 work with functions and the equation of a line.

8.EE.C.7 This is a culminating standard for solving one-variable linear equations.

8.EE.C.8 When students work toward meeting this standard, they build on what they know about two-variable linear equations, and they enlarge the varieties of real-world and mathematical problems they can solve.

8.F.A.2 Work toward meeting this standard repositions previous work with tables and graphs in the new context of input/output rules.

8.G.B.7 The Pythagorean theorem is useful in practical problems, relates to grade-level work in irrational numbers and plays an important role mathematically in coordinate geometry in high school.

Examples of Opportunities for Connecting Mathematical Content and Mathematical Practices

Mathematical practices should be evident *throughout* mathematics instruction and connected to all of the content areas highlighted above, as well as all other content areas addressed at this grade level. Mathematical tasks (short, long, scaffolded, and unscaffolded) are an important opportunity to connect content and practices. Some brief examples of how the content of this grade might be connected to the practices follow.

- When students convert a fraction such as $1/7$ to a decimal, they might notice that they are repeating the same calculations and conclude that the decimal repeats. Similarly, by repeatedly checking whether points are on a line through (1, 2) with slope 3, students might abstract the equation of the line in the form $(y - 2)/(x - 1) = 3$. In both examples, students look for and express regularity in repeated reasoning (MP.8).

- The Pythagorean theorem can provide opportunities for students to construct viable arguments and critique the reasoning of others (e.g., if a student in the class seems to be confusing the theorem with its converse) (MP.3).

- Solving an equation such as $3(x - 1/2) = x + 2$ requires students to see and make use of structure (MP.7).

- Much of the mathematics in grade 8 lends itself to modeling (MP.4). For example, standard 8.F.B.4 involves modeling linear relationships with functions.

- Scientific notation (8.EE.A.4) presents opportunities for strategically using appropriate tools (MP.5). For example, the computation $(1.73 \times 10^{-4}) \cdot (1.73 \times 10^{-5})$ can be done quickly with a calculator by squaring 1.73 and then using properties of exponents to determine the exponent of the product by inspection.

Content Emphases by Cluster[22]

Not all of the content in a given grade is emphasized equally in the standards. Some clusters require greater emphasis than the others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness. In addition, an intense focus on the most critical material at each grade allows depth in learning, which is carried out through the Standards for Mathematical Practice.

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[22] Refer to pages 13–14 for further explanation of the cluster-level emphases. Refer also to the Common Core State Standards for Mathematics for the standards that fall within each cluster.

Key: ■ Major Clusters; ▣ Supporting Clusters; ○ Additional Clusters

The Number System

▣ A. Know that there are numbers that are not rational, and approximate them by rational numbers.

Expressions and Equations

■ A. Work with radicals and integer exponents.

■ B. Understand the connections between proportional relationships, lines and linear equations.

■ C. Analyze and solve linear equations and pairs of simultaneous linear equations.

Functions

■ A. Define, evaluate and compare functions.

■ B. Use functions to model relationships between quantities.

Geometry

■ A. Understand congruence and similarity using physical models, transparencies or geometry software.

■ B. Understand and apply the Pythagorean Theorem.

○ C. Solve real-world and mathematical problems involving volume of cylinders, cones and spheres.

Statistics and Probability

▣ A. Investigate patterns of association in bivariate data.

Examples of Linking Supporting Clusters to the Major Work of the Grade

- Know that there are numbers that are not rational, and approximate them by rational numbers: Work with the number system in this grade (8.NS.A.1–2) is intimately related to work with radicals (8.EE.A.2), and both of these may be connected to the Pythagorean theorem (8.G, **second cluster**) as well as to volume problems (8.G.C.9), e.g., in which a cube has known volume but unknown edge lengths.

- Investigate patterns of association in bivariate data: Looking for patterns in scatterplots and using linear models to describe data are directly connected to the work in the Expressions and Equations clusters. Together, these represent a connection to the Standard for Mathematical Practice, MP.4: Model with mathematics.