Mixed Probability Practice

Name:

Consider the following frequency table.

x	Frequency
2	8
4	15
7	21
10	28
11	3

1a. *[1 mark]* Write down the mode.

1b. *[2 marks]* Find the value of the range.

1c. *[2 marks]* Find the mean.

1d. *[2 marks]* Find the variance.

2a. *[2 marks]* In a large university the probability that a student is left handed is 0.08. A sample of 150 students is randomly selected from the university. Let k be the expected number of left-handed students in this sample.Find k.

2b. *[2 marks]* Hence, find the probability that exactly k students are left handed;

2c. *[2 marks]* Hence, find the probability that fewer than k students are left handed.

3a. *[2 marks]* The following table shows a probability distribution for the random variable X, where $\mathrm{E}(X) = 1.2$. Find q.

x	0	1	2	3
$\mathrm{P}(X=x)$	p	$\frac{1}{2}$	$\frac{3}{10}$	q

3b. *[2 marks]* Find p.

3c. *[1 mark]* A bag contains white and blue marbles, with at least three of each colour. Three marbles are drawn from the bag, without replacement. The number of blue marbles drawn is given by the random variable X.

Write down the probability of drawing three blue marbles.

3d. *[1 mark]* Explain why the probability of drawing three white marbles is $\frac{1}{6}$.

3e. *[3 marks]* The bag contains a total of ten marbles of which w are white. Find w.

3f. *[4 marks]* A game is played in which three marbles are drawn from the bag of ten marbles, without replacement. A player wins a prize if three white marbles are drawn.

Grant plays the game until he wins two prizes. Find the probability that he wins his second prize on his eighth attempt.

4a. *[2 marks]* Events A and B are independent with $\mathrm{P}(A \cap B) = 0.2$ and $\mathrm{P}(A' \cap B) = 0.6$.

Find $\mathrm{P}(B)$.

4b. *[4 marks]* Find $\mathrm{P}(A \cup B)$.

5a. *[2 marks]* There are 10 items in a data set. The sum of the items is 60.

Find the mean.

5b. *[3 marks]* The variance of this data set is 3. Each value in the set is multiplied by 4.

(i) Write down the value of the new mean.

(ii) Find the value of the new variance.

6a. *[4 marks]* The following table shows the average number of hours per day spent watching television by seven mothers and each mother's youngest child.

Hours per day that a mother watches television (x)	2.5	3.0	3.2	3.3	4.0	4.5	5.8
Hours per day that her child watches television (y)	1.8	2.2	2.6	2.5	3.0	3.2	3.5

The relationship can be modelled by the regression line with equation $y = ax + b$.

(i) Find the correlation coefficient.

(ii) Write down the value of a and of b.

6b. *[3 marks]* Elizabeth watches television for an average of 3.7 hours per day.

Use your regression line to predict the average number of hours of television watched per day by Elizabeth's youngest child. Give your answer correct to one decimal place.

Mixed Probability Practice Name:

Consider the following frequency table.

x	Frequency
2	8
4	15
7	21
10	28
11	3

1a. *[1 mark]* Write down the mode. 10

1b. *[2 marks]* Find the value of the range. $11 - 2 = 9$

1c. *[2 marks]* Find the mean. 2.8 + 4 15 7.15

Lists + Spreadsheets.

1d. *[2 marks]* Find the variance. $\sigma^2 = 2.9^2 = 8.45$

2a. *[2 marks]* In a large university the probability that a student is left handed is 0.08. A sample of 150 students is randomly selected from the university. Let k be the expected number of left-handed students in this sample.Find k.

$k = 150 \;\; .08 = 12$

2b. *[2 marks]* Hence, find the probability that exactly k students are left handed;

$P(x=12) = $ $_{150}C_{12}(.08)^{12}(.92)^{138}$ $= .119$

2c. *[2 marks]* Hence, find the probability that fewer than k students are left handed.

$P(x \le 11) = P(x < 12) =$ binomcdf $(0, 11, 0.8) = .457$

3a. *[2 marks]* The following table shows a probability distribution for the random variable X, where $\mathrm{E}(X) = 1.2$. Find q.

$0p + \frac{1}{2} + \frac{6}{10} + 3q = 1.2$

$3q = 1.2 + 1.1$

$q = \frac{1}{30}$

Blue Marbles x	0	1	2	3
$\mathrm{P}(X = x)$	p	$\frac{1}{2}$ $\frac{15}{30}$	$\frac{3}{10}$ $\frac{9}{30}$	$\frac{1}{30}$ q

3b. *[2 marks]* Find p.

$\sum p(x) = 1 - \frac{25}{30} = \frac{5}{30} = \frac{1}{6} = .167$

3c. *[1 mark]* A bag contains white and blue marbles, with at least three of each colour. Three marbles are drawn from the bag, without replacement. The number of blue marbles drawn is given by the random variable X.

Write down the probability of drawing three blue marbles. $P(x=3) = \frac{1}{30}$

3d. *[1 mark]* Explain why the probability of drawing three white marbles is $\frac{1}{6}$. $P(x=0) = \frac{1}{6}$

b/c there's no blue marbles

$P(3W) = \frac{1}{6}$ $\frac{x}{10} \cdot \frac{(x-1)}{9} \cdot \frac{x-2}{8} = \frac{1}{6}$ n Solve

$w \cdot w \cdot w = \frac{1}{6}$

$x = 6$

3e. *[3 marks]* The bag contains a total of ten marbles of which w are white. Find w.

3f. *[4 marks]* A game is played in which three marbles are drawn from the bag of ten marbles, without replacement. A player wins a prize if three white marbles are drawn.

Grant plays the game until he wins two prizes. Find the probability that he wins his second prize on his eighth attempt.

${}_7C_1 \left(\frac{1}{6}\right)^1 \left(\frac{5}{6}\right)^6 \cdot \left(\frac{1}{6}\right)$

$.390714 \times \frac{1}{6} = .0651$

4a. *[2 marks]* Events A and B are independent with $P(A \cap B) = 0.2$ and $P(A' \cap B) = 0.6$.

Find $P(B)$. $.8$

$\boxed{.2 \quad .6}$

4b. *[4 marks]* Find $P(A \cup B)$.

$P(A) \cdot P(B) = .2$

$P(A) = \frac{.2}{.8} = \frac{1}{4} = .25$

5a. *[2 marks]* There are 10 items in a data set. The sum of the items is 60.

Find the mean. $\frac{60}{10} = 6$

$\sigma = \sqrt{3}$

5b. *[3 marks]* The variance of this data set is 3. Each value in the set is multiplied by 4.

(i) Write down the value of the new mean. $6 \times 4 = 24$

(ii) Find the value of the new variance. $(\sqrt{3} \times 4)^2 = 16 \cdot 3 = 48$ or $3 \times 4^2 = 48$

6a. *[4 marks]* The following table shows the average number of hours per day spent watching television by seven mothers and each mother's youngest child.

Hours per day that a mother watches television (x)	2.5	3.0	3.2	3.3	4.0	4.5	5.8
Hours per day that her child watches television (y)	1.8	2.2	2.6	2.5	3.0	3.2	3.5

The relationship can be modelled by the regression line with equation $y = ax + b$.

(i) Find the correlation coefficient. $r = .947$

(ii) Write down the value of a and of b. $a = .501$ $b = .804$

6b. *[3 marks]* Elizabeth watches television for an average of 3.7 hours per day.

Use your regression line to predict the average number of hours of television watched per day by Elizabeth's youngest child. Give your answer correct to one decimal place.

$y = .501(3.7) + .804$

$y = 2.7$

Mix Prob Test Practice

1a. *[6 marks]* The following table shows the probability distribution of a discrete random variable A, in terms of an angle θ.

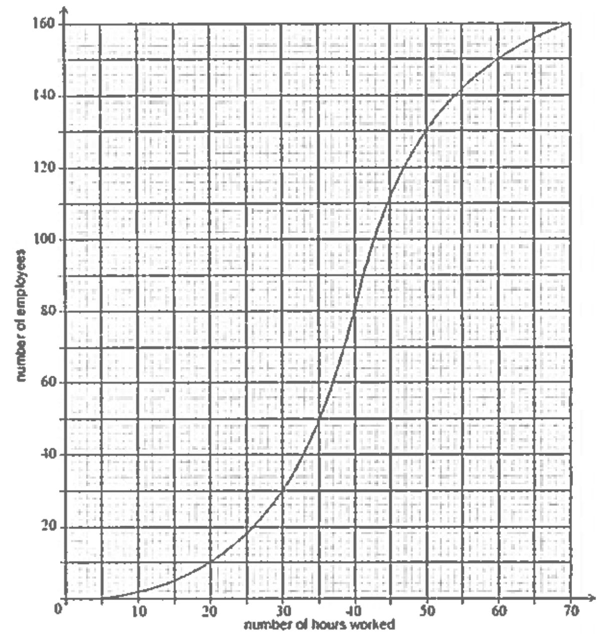
a	1	2
$P(A = a)$	$\cos\theta$	$2\cos 2\theta$

Show that $\cos\theta = \frac{3}{4}$.

1b. *[3 marks]* Given that $\tan\theta > 0$, find $\tan\theta$.

1c. *[6 marks]* Let $y = \frac{1}{\cos x}$, for $0 < x < \frac{\pi}{2}$. The graph of y between $x = \theta$ and $x = \frac{\pi}{4}$ is rotated 360° about the x-axis. Find the volume of the solid formed.

2a. *[2 marks]* A city hired 160 employees to work at a festival. The following cumulative frequency curve shows the number of hours employees worked during the festival.



Find the median number of hours worked by the employees.

2b. *[1 mark]* Write down the number of employees who worked 50 hours or less.

2c. *[1 mark]* The city paid each of the employees £8 per hour for the first 40 hours worked, and £10 per hour for each hour they worked after the first 40 hours. Find the amount of money an employee earned for working 40 hours;

2d. *[3 marks]* Find the amount of money an employee earned for working 43 hours.

2e. *[3 marks]* Find the number of employees who earned £200 or less.

2f. *[4 marks]* Only 10 employees earned more than £k. Find the value of k.

3a. *[2 marks]* Ten students were surveyed about the number of hours, x, they spent browsing the Internet during week 1 of the school year. The results of the survey are given below.

$$\sum_{i=1}^{10} x_i = 252,\ \sigma = 5 \text{ and median} = 27.$$

Find the mean number of hours spent browsing the Internet.

3b. *[2 marks]* During week 2, the students worked on a major project and they each spent an additional five hours browsing the Internet. For week 2, write down

(i) the mean;

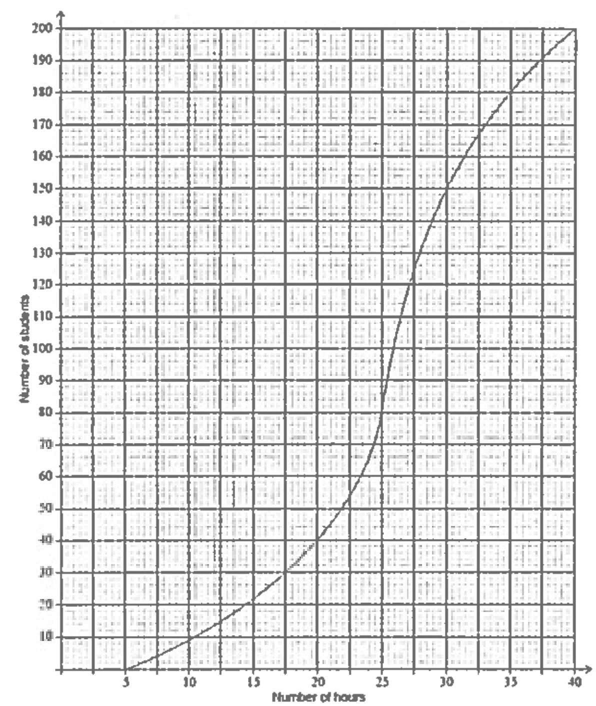
(ii) the standard deviation.

3c. *[6 marks]* During week 3 each student spent 5% less time browsing the Internet than during week 1. For week 3, find

(i) the median;

(ii) the variance.

3d. *[6 marks]* During week 4, the survey was extended to all 200 students in the school. The results are shown in the cumulative frequency graph:



(i) Find the number of students who spent between 25 and 30 hours browsing the Internet.

(ii) Given that 10% of the students spent more than k hours browsing the Internet, find the maximum value of k.

Mix Prob Test Practice

1a. *[6 marks]* The following table shows the probability distribution of a discrete random variable A, in terms of an angle θ.

a	1	2
$P(A = a)$	$\cos\theta$	$2\cos 2\theta$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$
$= 2\cos^2\theta - 1$
$= 1 - 2\sin^2\theta$

Show that $\cos\theta = \frac{3}{4}$.

$\cos\theta + 2(\cos 2\theta) = 1$
$\cos\theta + 2(2\cos^2\theta - 1) = 1$
$\cos\theta + 4\cos^2\theta - 2 = 1$

$\cos\theta + 4\cos^2\theta - 3 = 0$
$4\cos^2\theta + \cos\theta - 3 = 0$
$(4\cos\theta - 3)(\cos\theta + 1) = 0$

$\cos\theta = \frac{3}{4}$ $\cancel{\cos\theta = -1}$

prob is b/n $[0, 1]$

1b. *[3 marks]* Given that $\tan\theta > 0$, find $\tan\theta$.

$\sin^2 + \left(\frac{3}{4}\right)^2 = 1$

$1 - \frac{9}{16} = \sin^2\theta$

$\sqrt{\frac{7}{16}} = \sin^2\theta$

$\sin\theta = \frac{\sqrt{7}}{4}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

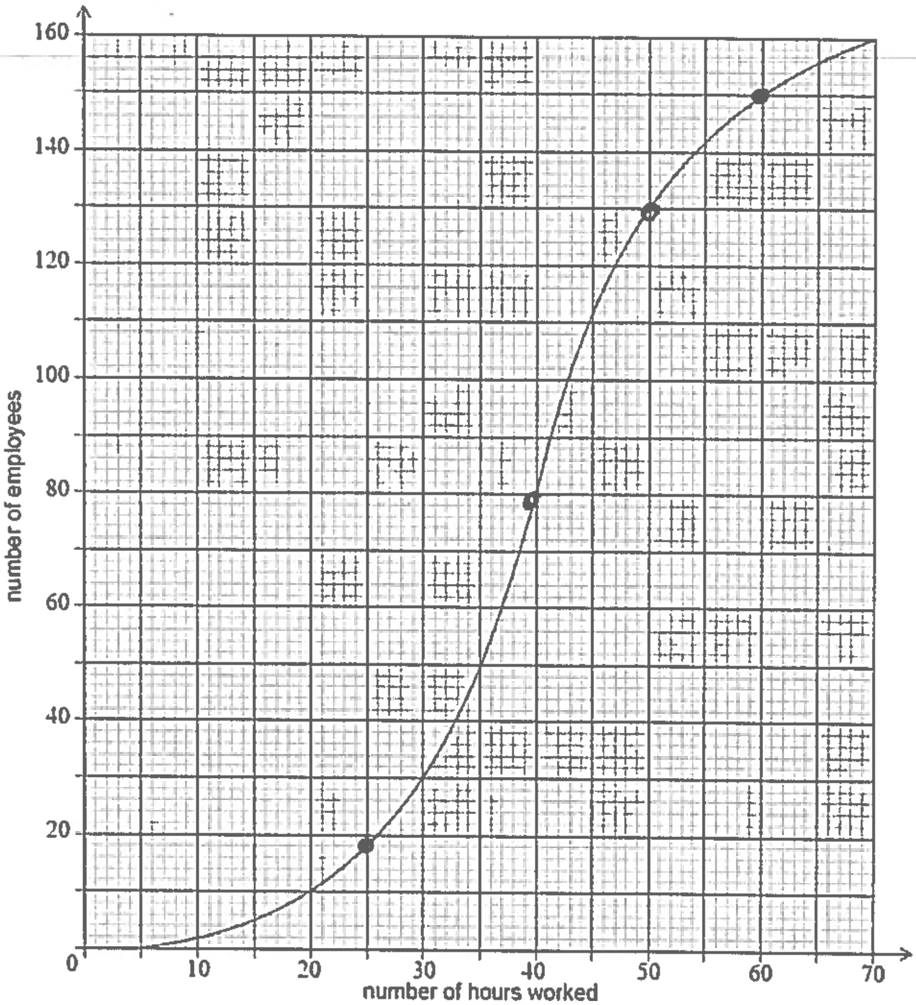
$\tan\theta = \frac{\sqrt{7}}{3}$

1c. *[6 marks]* Let $y = \frac{1}{\cos x}$, for $0 < x < \frac{\pi}{2}$. The graph of y between $x = \theta$ and $x = \frac{\pi}{4}$ is rotated 360° about the x-axis. Find the volume of the solid formed.

$\pi \int_{\theta}^{\frac{\pi}{4}} \left(\frac{1}{\cos x}\right)^2 dx = \pi \left(\tan\theta \Big|_{\theta}^{\pi/4} \right) = \pi\left(\tan\frac{\pi}{4} - \tan\theta\right)$

$= \pi\left(1 - \frac{\sqrt{7}}{3}\right)$

2a. *[2 marks]* A city hired 160 employees to work at a festival. The following cumulative frequency curve shows the number of hours employees worked during the festival.



Find the median number of hours worked by the employees. 80th employee

40

2b. *[1 mark]* Write down the number of employees who worked 50 hours or less.

130

2c. *[1 mark]* The city paid each of the employees £8 per hour for the first 40 hours worked, and £10 per hour for each hour they worked after the first 40 hours. Find the amount of money an employee earned for working 40 hours;

$40 \times 8 = 320$

2d. *[3 marks]* Find the amount of money an employee earned for working 43 hours.

$40 \times 8 + 10 \times 3 = 320 + 30 = 350$

2e. *[3 marks]* Find the number of employees who earned £200 or less.

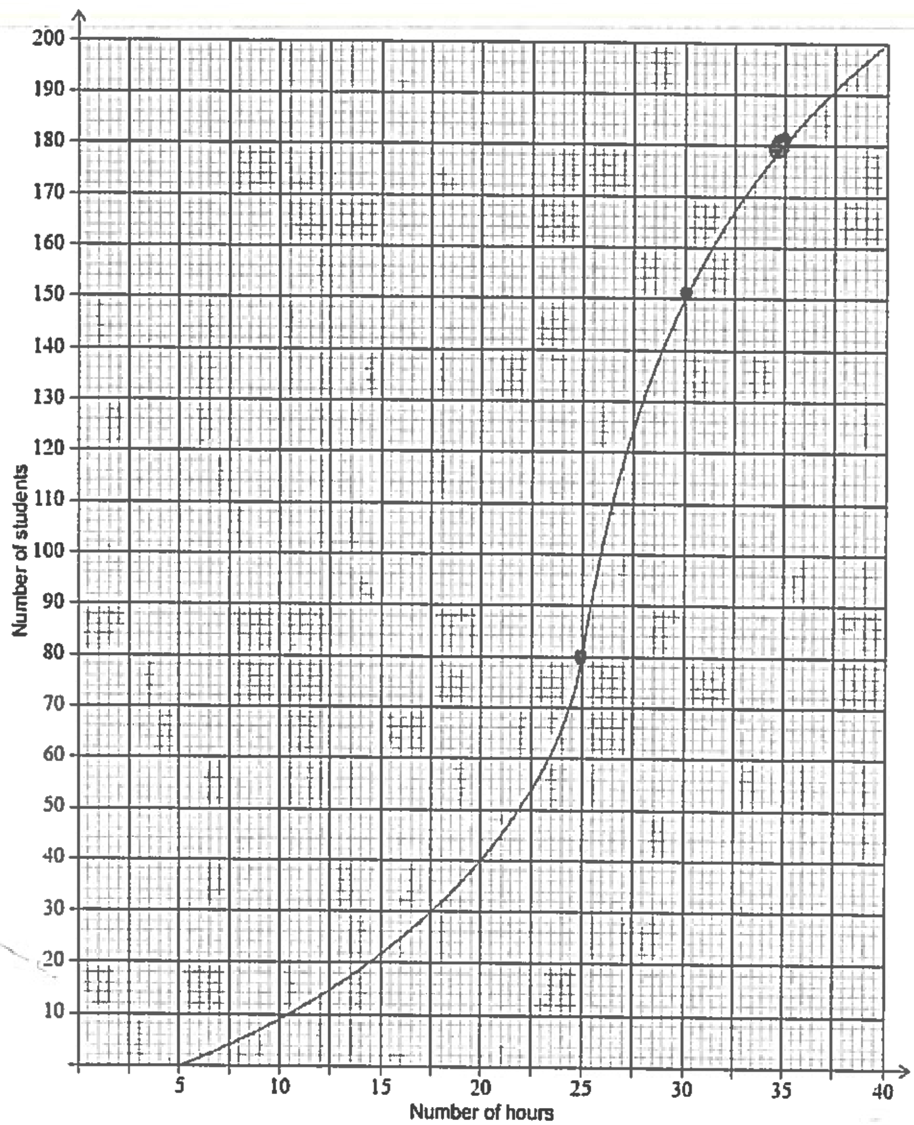
$\frac{200}{8} = 25$ hrs 18 employees

2f. *[4 marks]* Only 10 employees earned more than £k. Find the value of k.

$320 + 10 \times 20 = 320 + 200 = 520$

150 employees - 60 hrs

3d. *[6 marks]* During week 4, the survey was extended to all 200 students in the school. The results are shown in the cumulative frequency graph:



150 - 80 = 70

(i) Find the number of students who spent between 25 and 30 hours browsing the Internet.

(ii) Given that 10% of the students spent more than k hours browsing the Internet, find the maximum value of k.

10% = 20 students → 180 students

35 hrs

3a. *[2 marks]* Ten students were surveyed about the number of hours, x, they spent browsing the Internet during week 1 of the school year. The results of the survey are given below.

$$\sum_{i=1}^{10} x_i = 252,\ \sigma = 5 \text{ and median} = 27.$$

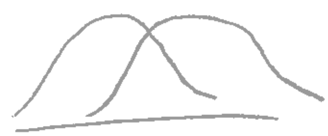
Find the mean number of hours spent browsing the Internet.

$\frac{252}{10} = 25.2$

3b. *[2 marks]* During week 2, the students worked on a major project and they each spent an additional five hours browsing the Internet. For week 2, write down

(i) the mean; 30.2

$25.2+5$



$\sigma = \sigma$

$x_1 \rightarrow x_2$

$\bar{x}+5$

(ii) the standard deviation. 5

3c. *[6 marks]* During week 3 each student spent 5% less time browsing the Internet than during week 1. For week 3, find

(i) the median; $27 \times .95 = 25.65$ or 25.7

exact

(ii) the variance. $(.95 \times 5)^2 = 4.75^2 = 22.5625$ or 22.6

exact

or $.95^2 \times 5^2$