Lesson 6: Using Tree Diagrams to Represent a Sample Space and to Calculate Probabilities

List of all possible outcomes.

Classwork

Suppose a girl attends a preschool where the students are studying primary colors. To help teach calendar skills, the teacher has each student maintain a calendar in his or her cubby. For each of the four days that they are covering primary colors in class, students get to place a colored dot on their calendar: blue, yellow, or red. When the four days of the school week have passed (Monday–Thursday), what might the young girl's calendar look like?

One outcome would be four blue dots if the student chose blue each day. But consider that the first day (Monday) could be blue, and the next day (Tuesday) could be yellow, and Wednesday could be blue, and Thursday could be red. Or, maybe Monday and Tuesday could be yellow, Wednesday could be blue, and Thursday could be red. Or, maybe Monday, Tuesday, and Wednesday could be blue, and Thursday could be red, and so on and so forth.

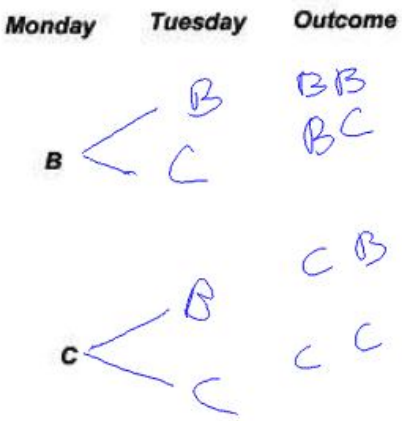
As hard to follow as this seems now, we have only mentioned 3 of the 81 possible outcomes in terms of the four days of colors! Listing the other 78 outcomes would take several pages! Rather than listing outcomes in the manner described above (particularly when the situation has multiple stages, such as the multiple days in the case above), we often use a *tree diagram* to display all possible outcomes visually. Additionally, when the outcomes of each stage are the result of a chance experiment, tree diagrams are helpful for computing probabilities.

Example 1: Two Nights of Games

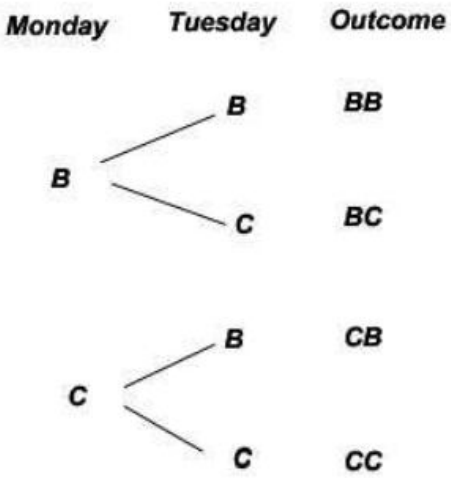
1 2 3 4

Imagine that a family decides to play a game each night. They all agree to use a tetrahedral die (i.e., a four-sided pyramidal die where each of four possible outcomes is equally likely—see image on page S.44) each night to randomly determine if they will play a board game (B) or a card game (C). The tree diagram mapping the possible overall outcomes over two consecutive nights will be developed below.

To make a tree diagram, first present all possibilities for the first stage. (In this case, Monday.)



Then, from *each* branch of the first stage, attach all possibilities for the second stage (Tuesday).



Note: If the situation has more than two stages, this process would be repeated until all stages have been presented.

a. If BB represents two straight nights of board games, what does CB represent?

CB is a card game on Mon. & a board game on Tues.

b. List the outcomes where exactly one board game is played over two days. How many outcomes were there?

BC, CB

2 outcomes

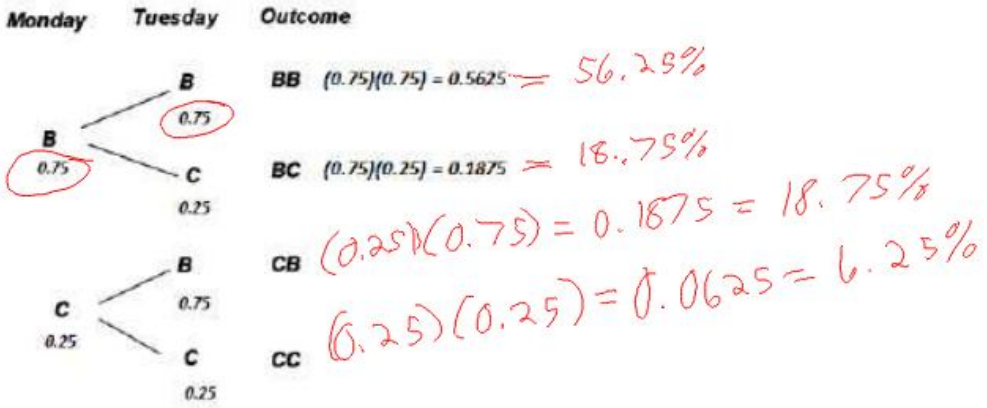
Card game | Board
1 | 2 3 4
$\frac{1}{4} = 0.25 = 25\%$ $\frac{3}{4} = 0.75 = 75\%$

Example 2: Two Nights of Games (with Probabilities)

In the example above, each night's outcome is the result of a chance experiment (rolling the tetrahedral die). Thus, there is a probability associated with each night's outcome.

By multiplying the probabilities of the outcomes from each stage, we can obtain the probability for each "branch of the tree." In this case, we can figure out the probability of each of our four outcomes: BB, BC, CB, and CC.

For this family, a card game will be played if the die lands showing a value of 1, and a board game will be played if the die lands showing a value of 2, 3, or 4. This makes the probability of a board game (B) on a given night 0.75.



a. The probabilities for two of the four outcomes are shown. Now, compute the probabilities for the two remaining outcomes.

16 outcomes

b. What is the probability that there will be exactly one night of board games over the two nights?

BC + CB
18.75% + 18.75% = 37.50%
0.1875 + 0.1875 = 0.3750