Lesson 4: Calculating Probabilities for Chance Experiments with Equally Likely Outcomes

Classwork

Examples: Theoretical Probability

In a previous lesson, you saw that to find an estimate of the probability of an event for a chance experiment you divide

$$P(\text{event}) = \frac{\text{Number of observed occurrences of the event}}{\text{Total number of observations}}.$$

Your teacher has a bag with some cubes colored yellow, green, blue, and red. The cubes are identical except for their color. Your teacher will conduct a chance experiment by randomly drawing a cube with replacement from the bag. Record the outcome of each draw in the table below.

Trial	Outcome
1	blue
2	green
3	red
4	blue
5	yellow
6	green
7	blue
8	red
9	green
10	blue
11	yellow
12	red
13	green
14	
15	
16	
17	
18	
19	
20	

y=4
g=6
r=5
b=5

1. Based on the 20 trials, estimate for the probability of

a. choosing a yellow cube.

$P(\text{yellow}) = \frac{4}{20} = \frac{1}{5}$

b. choosing a green cube.

$P(\text{green}) = \frac{6}{20} = \frac{3}{10}$

c. choosing a red cube.

$\frac{5}{20} = \frac{1}{4}$

d. choosing a blue cube.

$\frac{5}{20} = \frac{1}{4}$

2. If there are 40 cubes in the bag, how many cubes of each color are in the bag? Explain.

10 cubes of each color.

$\frac{1}{4}$ chance each color

3. If your teacher were to randomly draw another 20 cubes one at a time and with replacement from the bag, would you see exactly the same results? Explain.

No. The results will change for each set of drawings

4. Find the fraction of each color of cubes in the bag.

$\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{4}{4} = 1$ (Y, G, R, B; total cubes)

Yellow $\frac{10}{40} = \frac{1}{4}$

Green $\frac{10}{40} = \frac{1}{4}$

Red $\frac{10}{40} = \frac{1}{4}$

Blue $\frac{10}{40} = \frac{1}{4}$

Each fraction is the *theoretical probability* of choosing a particular color of cube when a cube is randomly drawn from the bag.

When all the possible outcomes of an experiment are equally likely, the probability of each outcome is

Same number of each color cubes

$$P(\text{outcome}) = \frac{1}{\text{Number of possible outcomes}}.$$

$P(\text{blue}) = \frac{1}{4}$

An event is a collection of outcomes, and when the outcomes are equally likely, the theoretical probability of an event can be expressed as

ex) 18 blue, 2 red > 20 total

$$P(\text{event}) = \frac{\text{Number of favorable outcomes}}{\text{Number of possible outcomes}}.$$

$P(\text{blue}) = \frac{18}{20} = \frac{9}{10}$

The theoretical probability of drawing a blue cube is

$$P(\text{blue}) = \frac{\text{Number of blue cubes}}{\text{Total number of cubes}} = \frac{10}{40}.$$

5. Is each color equally likely to be chosen? Explain your answer.

yes. Same number of cubes for each color.

6. How do the theoretical probabilities of choosing each color from Exercise 4 compare to the experimental probabilities you found in Exercise 1?

Theoretically all colors should have had a probability of $\frac{1}{4}$ but in our experiment we found green to have a probability of $\frac{3}{10}$. Yellow was $\frac{1}{5}$

7. An experiment consisted of flipping a nickel and a dime. The first step in finding the theoretical probability of obtaining a heads on the nickel and a heads on the dime is to list the sample space. For this experiment, the sample space is shown below.

Nickel	Dime
H	H
H	T
T	H
T	T

4 possibilities

If the counts are fair, these outcomes are equally likely, so the probability of each outcome is $\frac{1}{4}$.

Nickel	Dime	Probability
H	H	$\frac{1}{4}$
H	T	$\frac{1}{4}$
T	H	$\frac{1}{4}$
T	T	$\frac{1}{4}$

The probability of two heads is $\frac{1}{4}$ or $P(\text{two heads}) = \frac{1}{4}$.

Exercises

1. Consider a chance experiment of rolling a number cube.
   a. What is the sample space? List the probability of each outcome in the sample space.

Sample space: 1, 2, 3, 4, 5, 6

probability: $\frac{1}{6}$ for each number

   b. What is the probability of rolling an odd number?

1, 3, 5

$P(\text{odd}) = \frac{3}{6} = \frac{1}{2}$
$= 50\%$

   c. What is the probability of rolling a number less than 5?

1, 2, 3, 4

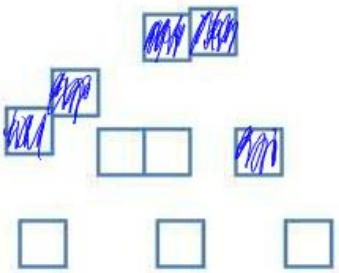
$P(<5) = \frac{4}{6} = \frac{2}{3}$

2. Consider an experiment of randomly selecting a letter from the word *number*.
   a. What is the sample space? List the probability of each outcome in the sample space.

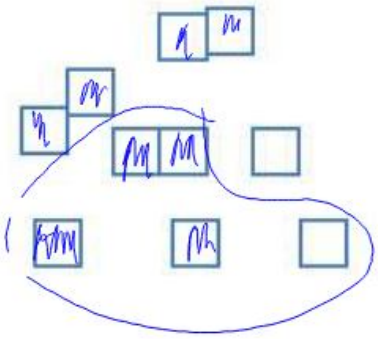
   b. What is the probability of selecting a vowel?

   c. What is the probability of selecting the letter z?

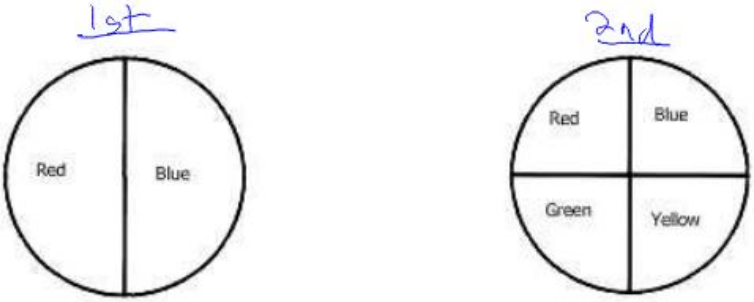
3. Consider an experiment of randomly selecting a cube from a bag of 10 cubes.
   a. Color the cubes below so that the probability of selecting a blue cube is $\frac{1}{2}$.



b. Color the cubes below so that the probability of selecting a blue cube is $\frac{4}{5}$. $= \frac{8}{10}$



4. Students are playing a game that requires spinning the two spinners shown below. A student wins the game if both spins land on red. What is the probability of winning the game? Remember to first list the sample space and the probability of each outcome in the sample space. There are eight possible outcomes to this chance experiment.



Sample space: RR, RB, RG, RY, BR, BB, BG, BY

Probability of each outcome: $\frac{1}{8}$