Chapter 5
Irrational Numbers

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Name ______________________ Period _____

Irrational Numbers

Date _______________ Notes

Lesson 1: Simplifying Radicals

What is a prime number?

A prime number is a number, other than 1, that can only be divided by its self and 1.

- Prime Numbers: 2, 13, 17, 23, 19, 11, 5, 7, 3

A radical or the principal n^{th} root of k: index Root → $\sqrt[n]{k}$ ← radicand

k, the radicand, is a real number.

n, the index, is a positive integer greater than one.

To simplify a radical means to find another expression with the same value. It does **NOT** mean to find a decimal approximation.

How do you simplify a radical?

$\sqrt{4}$ $\sqrt[3]{4}$

1. Identify n. (Remember if there is no number written in the place of the index/root, the index/root is 2.)
2. If the radicand contains a number, create a factor tree.
   - Break the number down into its prime numbers.
   - Rewrite all of the prime numbers in order from smallest to largest.
3. If the radicand contains a variable(s), write the variable as many times as the exponents says to.
4. If a number and/or variable appear the same number of times or more as the index/root, circle that number and/or variable the same number of times as the index/root. (i.e. If the index/root is 4 and 2 appears 6 times in the prime factorization, **ONLY** circle four 2s.)
5. The number(s) and/or variable(s) come out in front of the radical. (If there is a group of 2s, you place only ONE 2 in front.)
   - If there is already a number and/or variable in front, you multiply it by what you are taking out.
6. The number(s) and/or variable(s) that you did not circle, remain under the radical. If there is more than 1 number and/or variable, multiply them together under the radical.

Index tells us how many we need for a group

Examples

1. $\sqrt{54}$ $n=2$

$2\cdot3\cdot\boxed{3\cdot3}$

$=3\sqrt{2\cdot3}$

$=3\sqrt{6}$

2. $\sqrt[3]{320}$ $n=3$

$\boxed{2\cdot2\cdot2}\boxed{2\cdot2\cdot2}\,5$

$=2\cdot2\sqrt[3]{5}$

$=4\sqrt[3]{5}$

3. $\sqrt{a^4b^2}$ $n=2$

$\boxed{a\cdot a}\boxed{a\cdot a}\boxed{b\cdot b}$

$=a\cdot a\cdot b$

$=a^2b$

Practice:

$\sqrt[3]{48}=2\sqrt[3]{6}$

$n=3$

$\boxed{2\cdot2\cdot2}\,2\cdot3$

2

4. $\sqrt{18x^5y^4z}$ $n=2$

$2\,\boxed{3\cdot3}\,\boxed{x\cdot x}\,\boxed{x\cdot x}\,x\cdot\boxed{y\cdot y}\,\boxed{y\cdot y}\cdot z$

$=3x^2y^2\sqrt{2xz}$

5. $\sqrt[3]{54a^2b^5}$ $n=3$

$2\,\boxed{3\cdot3\cdot3}\,a\cdot a\,\boxed{b\cdot b\cdot b}\,b\cdot b$

$=3b\sqrt[3]{2a^2b^2}$

6. $\sqrt[3]{+40x^6}$ $n=3$

$\boxed{2\cdot2\cdot2}\,5\cdot\boxed{x\cdot x\cdot x}\,\boxed{x\cdot x\cdot x}$

$=2x^2\sqrt[3]{5}$

Practice:

$n=3$

$\sqrt[3]{750x^2y^4z}=5y\sqrt[3]{6x^2yz}$

$2\cdot3\,\boxed{5\cdot5\cdot5}\,x\cdot x\cdot\boxed{y\cdot y\cdot y}\,y\cdot z$

Practice:

7. $\sqrt[4]{162m^5n^2}$ $n=4$

$2\,\boxed{3\cdot3\cdot3\cdot3}\,\boxed{m\cdot m\cdot m\cdot m}\,m$

$n\cdot n$

$=3m\sqrt[4]{2mn^2}$

8. $-4\sqrt{216x^2y^2z}$ $n=2$

$\boxed{2\cdot2}\,2\,\boxed{3\cdot3}\,3\,\boxed{x\cdot x}\,\boxed{y\cdot y}\,z$

$2\cdot3\cdot x\cdot y$

$=-4\cdot6xy\sqrt{6z}$

$=-24xy\sqrt{6z}$

9. $5a\sqrt[4]{192a^6b^3}$ $n=4$

$\boxed{2\cdot2\cdot2\cdot2}\,2\cdot2\cdot3$

$\boxed{a\cdot a\cdot a\cdot a}\,a\cdot a\cdot b\cdot b\cdot b$

$=5a\cdot2a\sqrt[4]{12a^2b^3}$

$=10a^2\sqrt[4]{12a^2b^3}$

Practice:

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Homework:

1. $\sqrt{512x^2}$

2. $\sqrt[3]{24m^3}$

3. $\sqrt[3]{16a^3b^8}$

4. $6x\sqrt[3]{56x^5y}$

5. $16x^2y^2\sqrt[4]{128x^7y^7}$